INSTRUCTOR'S SOLUTIONS MANUAL

MATTHEW G. HUDELSON

BASIC TECHNICAL MATHEMATICS

AND

BASIC TECHNICAL MATHEMATICS WITH CALCULUS

ELEVENTH EDITION

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Instructor's Solutions Manual for
Basic Technical Mathematics and Basic Technical Mathematics with Calculus, 11th Edition

Chapter 1

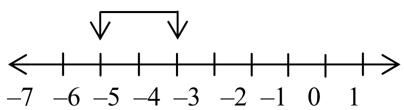
Basic Algebraic Operations

1.1 Numbers

1. The numbers –7 and 12 are integers. They are also rational numbers since they can be written as $\frac{-7}{1}$ and $\frac{12}{1}$.

2. The absolute value of –6 is 6, and the absolute value of –7 is 7. We write these as $|-6| = 6$ and $|-7| = 7$.

3. $-6 < -4$; –6 is to the left of –4.



4. The reciprocal of $\frac{3}{2}$ is $\frac{1}{3/2} = 1 \times \frac{2}{3} = \frac{2}{3}$.

5. 3 is an integer, rational $\left(\frac{3}{1}\right)$, and real.

$\sqrt{-4}$ is imaginary.

6. $\frac{\sqrt{7}}{3}$ is irrational (because $\sqrt{7}$ is an irrational number) and real.

-6 is an integer, rational $\left(\frac{-6}{1}\right)$, and real.

7. $-\frac{\pi}{6}$ is irrational (because π is an irrational number) and real.

$\frac{1}{8}$ is rational and real.

8. $-\sqrt{-6}$ is imaginary.

$-2.33 = \frac{-233}{100}$ is rational and real.

9. $|3| = 3$

$|-3| = 3$

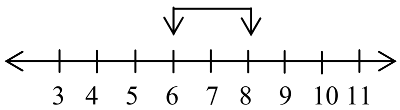
$\left|-\frac{\pi}{2}\right| = \frac{\pi}{2}$

10. $|-0.857| = 0.857$

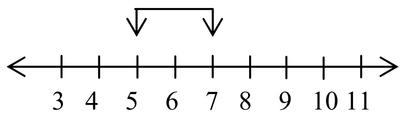
$\left|\sqrt{2}\right| = \sqrt{2}$

$\left|-\frac{19}{4}\right| = \frac{19}{4}$

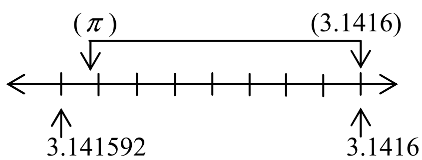
11. $6 < 8$; 6 is to the left of 8.



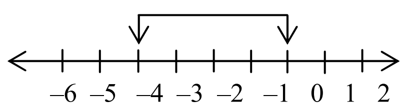
12. $7 > 5$; 7 is to the right of 5.



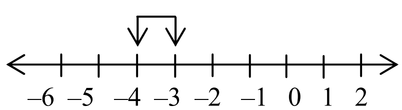
13. $\pi < 3.1416$; π $(3.1415926\ldots)$ is to the left of 3.1416.



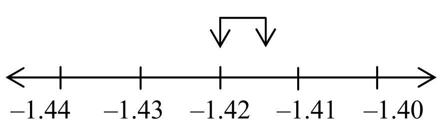
14. $-4 < 0$; -4 is to the left of 0.



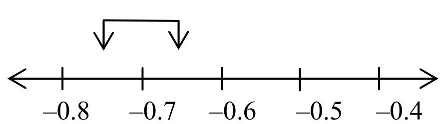
15. $-4 < -|-3|$; -4 is to the left of $-|-3|$, $\left(-|-3| = -(3) = -3\right)$.



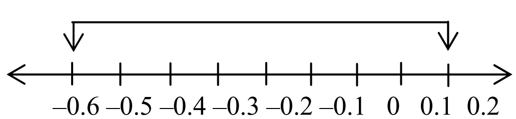
16. $-\sqrt{2} > -1.42$; $(-\sqrt{2} = -(1.414\ldots.) = -1.414\ldots), -\sqrt{2}$ is to the right of -1.42.



17. $-\frac{2}{3} > -\frac{3}{4}; -\frac{2}{3} = -0.666\ldots$ is to the right of $-\frac{3}{4} = -0.75$.



18. $-0.6 < 0.2$; -0.6 is to the left of 0.2.



19. The reciprocal of 3 is $\frac{1}{3}$.

The reciprocal of $-\frac{4}{\sqrt{3}}$ is $-\frac{1}{4/\sqrt{3}} = -\frac{\sqrt{3}}{4}$.

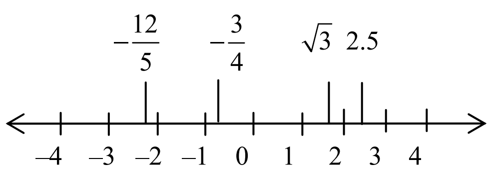
The reciprocal of $\frac{y}{b}$ is $\frac{1}{y/b} = \frac{b}{y}$.

20. The reciprocal of $-\frac{1}{3}$ is $-\frac{1}{1/3} = -\frac{3}{1} = -3$.

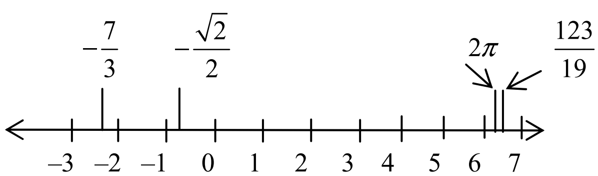
The reciprocal of $0.25 = \frac{1}{4}$ is $\frac{1}{1/4} = \frac{4}{1} = 4$.

The reciprocal of $2x$ is $\frac{1}{2x}$.

21. Find 2.5, $-\frac{12}{5} = -2.4; -\frac{3}{4} = -0.75; \sqrt{3} = 1.732...$



22. Find $-\frac{7}{3} = -2.333...; -\frac{\sqrt{2}}{2} = -\frac{1.414\ldots}{2} = -0.707;\ 2\pi = 2\times 3.14\ldots = 6.28;\ \frac{123}{19} = 6.47$.



23. An absolute value is not always positive, $|0| = 0$ which is not positive.

24. Since $-2.17 = -\frac{217}{100}$, it is rational.

25. The reciprocal of the reciprocal of any positive or negative number is the number itself. The reciprocal of n is $\frac{1}{n}$; the reciprocal of $\frac{1}{n}$ is $\frac{1}{1/n} = 1 \cdot \frac{n}{1} = n$.

26. Any repeating decimal is rational, so $2.\overline{72}$ is rational. It turns out that $2.\overline{72} = \frac{30}{11}$.

27. It is true that any nonterminating, nonrepeating decimal is an irrational number.

28. No, $|b-a| = |b| - |a|$, as shown below.

If $a > 0$, then $|a| = a$.

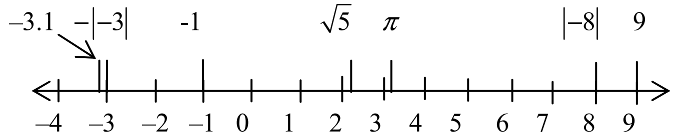
If $b > a$ and $a > 0$, then $|b| = b$.

If b $> a$ then $b - a > 0$, then $|b-a| = b - a$.

Therefore, $|b-a| = b - a = |b| - |a|$.

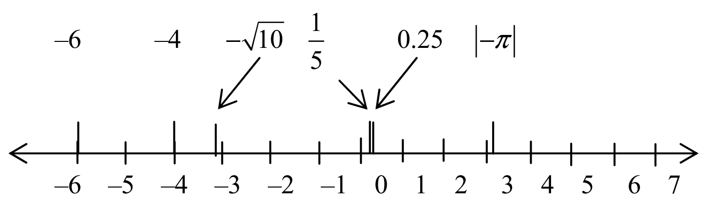
The two sides of the expression are equivalent, one side is not less than the other.

29. List these numbers from smallest to largest: $-1,\ 9,\ \pi = 3.14,\ \sqrt{5} = 2.236,\ |-8| = 8,\ -|-3| = -3,\ -3.1$.



So, from smallest to largest, they are $-3.1,\ -|-3|,\ -1,\ \sqrt{5},\ \pi,\ |-8|,\ 9$.

30. List these numbers from smallest to largest: $\frac{1}{5} = 0.20,\ -\sqrt{10} = -3.16...,\ -|-6| = -6,\ -4,\ 0.25,\ |-\pi| = 3.14...$.



So, from smallest to largest, they are $-|-6|,\ -4,\ -\sqrt{10},\ \frac{1}{5},\ 0.25,\ |-\pi|$.

31. If a and b are positive integers and $b > a$, then

(a) $b - a$ is a positive integer.

(b) $a - b$ is a negative integer.

(c) $\frac{b-a}{b+a}$, the numerator and denominator are both positive, but the numerator is less than the denominator, so the answer is a positive rational number than is less than 1.

32. If a and b are positive integers, then

(a) $a + b$ is a positive integer

(b) a / b is a positive rational number

(c) $a \times b$ is a positive integer

33. **(a)** Is the absolute value of a positive or a negative integer always an integer?

$|x| = x$, so the absolute value of a positive integer is an integer.

$|-x| = x$, so the absolute value of a negative integer is an integer.

(b) Is the reciprocal of a positive or negative integer always a rational number?

If x is a positive or negative integer, then the reciprocal of x is $\frac{1}{x}$. Since both 1 and x are integers, the reciprocal is a rational number.

34. **(a)** Is the absolute value of a positive or negative rational number rational?

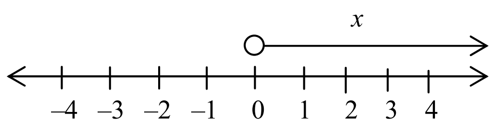
$|x| = x$, so if x is a positive or negative rational number, the absolute value of it is also a rational number.

(b) Is the reciprocal of a positive or negative rational number a rational number?

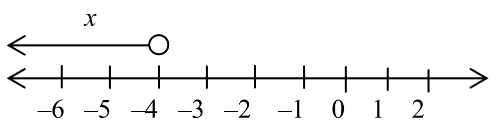
A rational number is a number that can be expressed as a fraction where both the numerator and denominator are integers and the denominator is not zero. So a rational number $\frac{\text{integer } a}{\text{integer } b}$ has a reciprocal of

$\frac{1}{\frac{\text{integer } a}{\text{integer } b}} = \frac{\text{integer } b}{\text{integer } a}$, which is also a rational number if integer a is not zero.

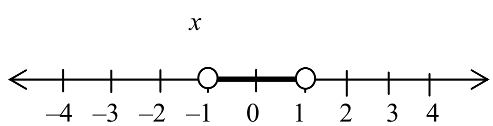
35. **(a)** If $x > 0$, then x is a positive number located to the right of zero on the number line.



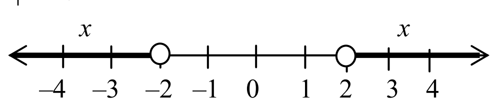
(b) If $x < -4$, then x is a negative number located to the left of –4 on the number line.



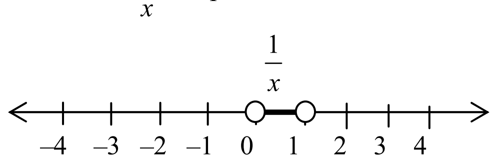
36. **(a)** If $|x| < 1$, then $-1 < x < 1$.



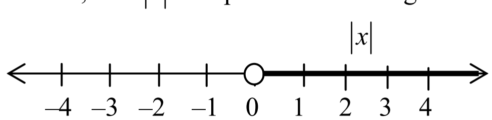
(b) $|x| > 2$, then $x < -2$ or $x > 2$.



37. If $x > 1$, then $\frac{1}{x}$ is a positive number less than 1. Or $0 < \frac{1}{x} < 1$.



38. If $x < 0$, then $|x|$ is a positive number greater than zero.



39. $a + bj = a + b\sqrt{-1}$ is a real number when $\sqrt{-1}$ is eliminated, which is when $b = 0$. So $a + bj$ is a real number for all real values of a and $b = 0$.

40. The variables are w and t.
The constants are c, 0.1, and 1.

41. $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$. Find C_T, where $C_1 = 0.0040\text{ F}$ and $C_2 = 0.0010\text{ F}$.

$$\frac{1}{C_T} = \frac{1}{0.0040} + \frac{1}{0.0010}$$

$$\frac{1}{C_T} = \frac{1(0.0040) + 1(0.0010)}{0.0040 \times 0.0010}$$

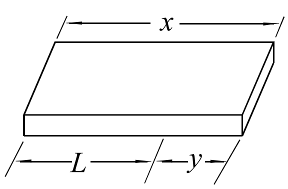
$$C_T = \frac{0.0040 \times 0.0010}{0.0040 + 0.0010} = \frac{0.0000040}{0.0050}$$

$$C_T = 0.00080 \text{ F}$$

42. $|100V| = 100V$
$|-200V| = 200V$
$|-200V| > |100V|$

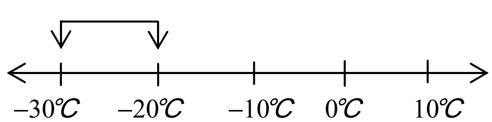
43. $N = \frac{a \text{ bits}}{\text{bytes}} \times \frac{1000 \text{ bytes}}{1 \text{ kilobyte}} \times n \text{ kilobytes}$
$N = 1000\, an$ bits

44.



$x =$ length of base in m
$y =$ the shortened length in centimetres.
$100x =$ length of base in cm
$y + L = 100x,$ all dimensions in cm
$L = 100x - y$

45. Yes, $-20\text{ °C} > -30\text{ °C}$ because -30 °C is found to the left of -20 °C on the number line.



46. For $I < 4$ A, $R > 12\ \Omega$.

1.2 Fundamental Operations of Algebra

1. $16-2\times(-2)=16-(-4)=16+4=20$

2. $\frac{-18}{-6}+5-(-2)(3)=3+5-(-6)=8+6=14$

3. $\frac{-12}{8-2}+\frac{5-1}{2(-1)}=\frac{-12}{6}+\frac{4}{-2}=-2+(-2)=-4$

4. $\frac{7\times 6}{0\times 0}=\frac{42}{0}=$ is undefined , not indeterminate.

5. $5+(-8)=5-8=-3$

6. $-4+(-7)=-4-7=-11$

7. $-3+9=6$ or alternatively
$-3+9=+(9-3)=+(6)=6$

8. $18-21=-3$ or alternatively
$18-21=-(21-18)=-(3)=-3$

9. $-19-(-16)=-19+16=-3$

10. $-8-(-10)=-8+10=2$

11. $7(-4)=-(7\times 4)=-28$

12. $-9(3)=-27$

13. $-7(-5)=+(7\times 5)=35$

14. $\frac{-9}{3}=-3$

15. $\frac{-6(20-10)}{-3}=\frac{-6(10)}{-3}=\frac{-60}{-3}=20$

16. $\frac{-28}{-7(5-6)}=\frac{-28}{-7(-1)}=\frac{-28}{7}=-4$

17. $-2(4)(-5)=-8(-5)=40$

18. $-3(-4)(-6)=12(-6)=-72$

19. $2(2-7)\div 10 = 2(-5)\div 10 = -10\div 10 = -1$

20. $\dfrac{-64}{-2|4-8|} = \dfrac{-64}{-2|-4|} = \dfrac{-64}{-2(4)} = \dfrac{-64}{-8} = 8$

21. $16\div 2(-4) = 8(-4) = -32$

22. $-20\div 5(-4) = -4(-4) = 16$

23. $-9-|2-10| = -9-|-8| = -9-8 = -17$

24. $(7-7)\div(5-7) = 0\div(-2) = 0$

25. $\dfrac{17-7}{7-7} = \dfrac{10}{0}$ is undefined

26. $\dfrac{(7-7)(2)}{(7-7)(-1)} = \dfrac{0(2)}{0(-1)} = \dfrac{0}{0}$ is indeterminate

27. $8-3(-4) = 8+12 = 20$

28. $-20+8\div 4 = -20+2 = -18$

29. $-2(-6)+\left|\dfrac{8}{-2}\right| = 12+|-4| = 12+4 = 16$

30. $\dfrac{|-2|}{-2} = \dfrac{2}{-2} = -1$

31.
$$\begin{aligned} 10(-8)(-3)\div(10-50) &= 10(-8)(-3)\div(-40) \\ &= -80(-3)\div(-40) \\ &= 240\div(-40) \\ &= -6 \end{aligned}$$

32. $\dfrac{7-|-5|}{-1(-2)} = \dfrac{7-5}{2} = \dfrac{2}{2} = 1$

33. $\dfrac{24}{3+(-5)} - 4(-9) = \dfrac{24}{-2} + (4\times 9) = -12+36 = 24$

34. $\dfrac{-18}{3} - \dfrac{4-|-6|}{-1} = \dfrac{-18}{3} - \dfrac{4-6}{-1} = -6 - \dfrac{-2}{-1} = -6-2 = -8$

35. $-7-\dfrac{|-14|}{2(2-3)}-3|6-8| = -7-\dfrac{14}{2(-1)}-3|-2|$

$= -7-\dfrac{14}{-2}-3(2)$

$= -7-(-7)-6$

$= -7+7-6$

$= -6$

36. $-7(-3)+\dfrac{-6}{-3}-|-9| = +(7\times 3)+2-9$

$= 21+2-9$

$= 14$

37. $\dfrac{3|-9-2(-3)|}{1-10} = \dfrac{3|-9+6|}{-9}$

$= \dfrac{3|-3|}{-9}$

$= \dfrac{9}{-9}$

$= -1$

38. $\dfrac{20(-12)-40(-15)}{98-|-98|} = \dfrac{-240+600}{98-98} = \dfrac{360}{0} =$ is undefined

39. $6(7)=(7)6$ demonstrates the commutative law of multiplication.

40. $6+8=8+6$ demonstrates the commutative law of addition.

41. $6(3+1)=6(3)+6(1)$ demonstrates the distributive law.

42. $4(5\times\pi)=(4\times 5)\pi$ demonstrates the associative law of multiplication.

43. $3+(5+9)=(3+5)+9$ demonstrates the associative law of addition.

44. $8(3-2)=8(3)-8(2)$ demonstrates the distributive law.

45. $(\sqrt{5}\times 3)\times 9=\sqrt{5}\times(3\times 9)$ demonstrates the associative law of multiplication.

46. $(3\times 6)\times 7=7\times(3\times 6)$ demonstrates the commutative law of multiplication.

47. $-a+(-b)=-a-b$, which is expression (d).

48. $b-(-a)=b+a=a+b$, which is expression (a).

49. $-b-(-a)=-b+a=a-b$, which is expression (b).

50. $-a-(-b)=-a+b=b-a$, which is expression (c).

51. Since $|5-(-2)|=|5+2|=|7|=7$ and $|-5-(-2)|=|-5+2|=|-3|=3$,
$|5-(-2)|>|-5-(-2)|$.

52. Since $|-3-|-7||=|-3-7|=|-10|=10$ and $||-3|-7|=|3-7|=|-4|=4$,
$|-3-|-7|| > ||-3|-7|$.

53. **(a)** The sign of a product of an even number of negative numbers is positive. Example: $-3(-6)=18$

(b) The sign of a product of an odd number of negative numbers is negative.
Example: $-5(-4)(-2)=-40$

54. Subtraction is not commutative because $x-y\neq y-x$. Example: $7-5=2$ does not equal $5-7=-2$

55. Yes, from the definition in Section 1.1, the absolute value of a positive number is the number itself, and the absolute value of a negative number is the corresponding positive number. So for values of x where $x>0$ (positive) or $x=0$ (neutral) then $|x|=x$.

Example: $|4|=4$.

The claim that absolute values of negative numbers $|x|=-x$ is also true.

Example: if x is -6, then $|-6|=-(-6)=6$.

56. The incorrect answer was achieved by subtracting before multiplying or dividing which violates the order of operations.
$24-6\div2\times3\neq18\div2\times3=9\times3=27$
The correct value is:
$24-6\div2\times3=24-3\times3=24-9=15$

57. **(a)** $-xy=1$ is true for values of x and y that are negative reciprocals of each other or $y=-\frac{1}{x}$, providing that the number x in the denominator is not zero. So if $x=12$, then $y=-\frac{1}{12}$ and $-xy=-(12)\left(-\frac{1}{12}\right)=1$.

(b) $\frac{x-y}{x-y}=1$ is true for all values of x and y, providing that $x\neq y$ to prevent division by zero.

58. **(a)** $|x+y|=|x|+|y|$ is true for values where both x and y have the same sign or either are zero:

$|x+y|=|x|+|y|$, when $x\geq0$ and $y\geq0$ or when $x\leq0$ and $y\leq0$

Example:

$|6+3|=6+3=9$ and

$|6|+|3|=6+3=9$

Also,

$|-6+(-3)|=|-9|=9$

$|-6|+|-3|=6+3=9$

$|x+y|=|x|+|y|$ is not true however, when x and y have opposite signs

$|x+y|\neq|x|+|y|$, when $x>0$ and $y<0$; or $x<0$ and $y>0$.

Example:
$|-21+6|=|-15|=15,$
$|-21|+|6|=21+6=27\neq 15$

$|4+(-5)|=|-1|=1,$
$|4|+|-5|=4+5=9\neq 1$

(b) In order for $|x-y|=|x|+|y|$ it is necessary that they have opposite signs or either to be zero.
Symbolically, $|x-y|=|x|+|y|$ when $x\geq 0$ and $y\leq 0$; or when $x\leq 0$ and $y\geq 0$.

Example:
$|6-(-3)|=6+3=9$ and
$|6|+|-3|=6+3=9$

Example:
$|-11-7|=|-18|=18$
$|-11|+|-7|=11+7=18$

$|x-y|=|x|+|y|$ is not true, however, when x and y have the same signs.
$|x-y|\neq|x|+|y|$, when $x>0$ and $y>0$; or $x<0$ and $y<0$.
Example:
$|21-6|=|15|=15,$
$|21|+|6|=27\neq 15$

59. The total change in the price of the stock is $-0.68+0.42+0.06+(-0.11)+0.02=-0.29$.

60. The difference in altitude is $-86-(-1396)=1396-86=1310$ m

61. The change in the meter energy reading E would be:
$E_{change}=E_{used}-E_{generated}$
$E_{change}=2.1\text{ kW}\cdot\text{h}-1.5\text{ kW}\left(3.0\text{ h}\right)$
$E_{change}=2.1\text{ kW}\cdot\text{h}-4.5\text{ kW}\cdot\text{h}$
$E_{change}=-2.4\text{ kW}\cdot\text{h}$

62. Assuming that this batting average is for the current season only which is just starting, the number of hits is zero and the total number of at-bats is also zero giving us a $\text{batting average}=\dfrac{\text{number of hits}}{\text{at}-\text{bats}}=\dfrac{0}{0}$ which is indeterminate, not 0.000.

63. The average temperature for the week is:
$$T_{avg}=\frac{-7+(-3)+2+3+1+(-4)+(-6)}{7}\ ^\circ\text{C}$$
$$T_{avg}=\frac{-7-3+2+3+1-4-6}{7}\ ^\circ\text{C}$$
$$T_{avg}=\frac{-14}{7}\ ^\circ\text{C}=-2.0\ ^\circ\text{C}$$

64. The vertical distance from the flare gun is
$d=(70)(5)+(-16)(25)$
$d=350+(-400)$
$d=350-400$
$d=-50 \text{ m}$
The flare is 50 m below the flare gun.

65. The sum of the voltages is
$V_{sum}=6\text{V}+(-2\text{V})+8\text{V}+(-5\text{V})+3\text{V}$
$V_{sum}=6\text{V}-2\text{V}+8\text{V}-5\text{V}+3\text{V}$
$V_{sum}=10\text{V}$

66. **(a)** The change in the current for the first interval is the second reading – the first reading
$Change_1=-2 \text{ lb/in}^2-7 \text{ lb/in}^2=-9 \text{ lb/in}^2$.
(b) The change in the current for the middle intervals is the third reading – the second reading
$Change_2=-9 \text{ lb/in}^2-(-2 \text{ lb/in}^2)=-9 \text{ lb/in}^2+2 \text{ lb/in}^2=-7 \text{ lb/in}^2$.
(c) The change in the current for the last interval is the last reading – the third reading
$Change_3=-6 \text{ lb/in}^2-(-9 \text{ lb/in}^2)=-6 \text{ lb/in}^2+9 \text{ lb/in}^2=3 \text{ lb/in}^2$.

67. The oil drilled by the first well is $100 \text{ m}+200 \text{ m}=300 \text{ m}$ which equals the depth drilled by the second well $200 \text{ m}+100 \text{ m}=300 \text{ m}$.
$100 \text{ m}+200 \text{ m}=200 \text{ m}+100 \text{ m}$ demonstrates the commutative law of addition.

68. The first tank leaks $12\frac{\text{L}}{\text{h}}(7 \text{ h})=84 \text{ L}$ .The second tank leaks $7\frac{\text{L}}{\text{h}}(12\text{h})=84\text{L}$.
$12\times 7=7\times 12$ demonstrates the commutative law of multiplication.

69. The total time spent browsing these websites is the total time spent browsing the first site on each day + the total time spent browsing the second site on each day
$t=7 \text{ days}\times 25\,\frac{\text{minutes}}{\text{day}}+7 \text{ days}\times 15\,\frac{\text{minutes}}{\text{day}}$
$t=175 \text{ min}+105 \text{ min}$
$t=280 \text{ min}$
OR
$t=7 \text{ days}\times(25+15)\,\frac{\text{minutes}}{\text{day}}$
$t=7 \text{ days}\times 40\,\frac{\text{minutes}}{\text{day}}$
$t=280 \text{ min}$
which illustrates the distributive law.

70. Distance = rate × time

$$d = \quad 600\frac{\text{km}}{\text{h}} + 50\frac{\text{km}}{\text{h}} \quad 3\text{ h}$$

$$d = 600\frac{\text{km}}{\text{h}}(3\text{h}) + 50\frac{\text{km}}{\text{h}}(3\text{h})$$

$$d = 1800\text{ km} + 150\text{ km} = 1950\text{ km}$$

OR

$$d = \quad 600\frac{\text{km}}{\text{h}} + 50\frac{\text{km}}{\text{h}} \quad 3\text{ h}$$

$$d = \quad 650\frac{\text{km}}{\text{h}} \quad 3\text{ h}$$

$$d = 1950\text{ km}$$

This illustrates the distributive law.

1.3 Calculators and Approximate Numbers

1. 0.390 has three significant digits since the zero is after the decimal. The zero is not necessary as a placeholder and should not be written unless it is significant.

2. 35.303 rounded off to four significant digits is 35.30.

3. In finding the product of the approximate numbers, $2.5 \times 30.5 = 76.25$, but since 2.5 has 2 significant digits, the answer is 76.

4. $38.3 - 21.9(-3.58) = 116.702$ using exact numbers; if we estimate the result, $40 - 20(-4) = 120$.

5. 8 cylinders is exact because they can be counted. 55 km/h is approximate since it is measured.

6. 0.002 mm thick is a measurement and is therefore an approximation. $7.50 is an exact price.

7. 24 hr and 1440 min (60 min/h × 24 h =1140 min) are both exact numbers.

8. 50 keys is exact because you can count them; 50 h of use is approximate since it is a measurement of time.

9. Both 1 cm and 9 g are measured quantities and so they are approximate.

10. The numbers 90 and 75 are exact counts of windows while 15 years is a measurement of time, hence it is approximate.

11. 107 has 3 significant digits; 3004 has 4 significant digits; 1040 has 3 significant digits (the final zero is a placeholder.)

12. 3600 has 2 significant digits; 730 has 2 significant digits; 2055 has 4 significant digits.

13. 6.80 has 3 significant digits since the zero indicates precision; 6.08 has 3 significant digits; 0.068 has 2 significant digits (the zeros are placeholders.)

14. 0.8730 has 4 significant digits; 0.0075 has 2 significant digits; 0.0305 has 3 significant digits.

15. 3000 has 1 significant digit; 3000.1 has 5 significant digits; 3000.10 has 6 significant digits.

16. 1.00 has 3 significant digits since the zeros indicate precision; 0.01 has 1 significant digit since leading zeros are not significant; 0.0100 has 3 significant digits, counting the trailing zeros.

17. 5000 has 1 significant digit; 5000.0 has 5 significant digits; $500\bar{0}$ has 4 significant digits since the bar over the final zero indicates that it is significant.

18. 200 has 1 significant digit; $20\bar{0}$ has 3 significant digits; 200.00 has 5 significant digits.

19. **(a)** 0.010 has more decimal places (3) and is more precise.
(b) 30.8 has more significant digits (3) and is more accurate.

20. **(a)** Both 0.041 and 7.673 have the same precision as they have the same number of decimal places (3).
(b) 7.673 is more accurate because it has more significant digits (4) than 0.041, which has 2 significant digits.

21. **(a)** Both 0.1 and 78.0 have the same precision as they have the same number of decimal places.
(b) 78.0 is more accurate because it has more significant digits (3) than 0.1, which has 1 significant digit.

22. **(a)** 0.004 is more precise because it has more decimal places (3).
(b) 7040 is more accurate because it has more significant digits (3) than 0.004, which has only 1 significant digit.

23. **(a)** 0.004 is more precise because it has more decimal places (3).
(b) Both have the same accuracy as they both have 1 significant digit.

24. The precision and accuracy of $|-8.914|$ and 8.914 are the same.
(a) Both 50.060 and 8.914 have the same precision as they have the same number of decimal places (3).
(b) 50.060 is more accurate because it has more significant digits (5) than 8.914, which has 4 significant digits.

25. **(a)** 4.936 rounded to 3 significant digits is 4.94.
(b) 4.936 rounded to 2 significant digits is 4.9.

26. **(a)** 80.53 rounded to 3 significant digits is 80.5.
(b) 80.53 rounded to 2 significant digits is 81.

27. **(a)** -50.893 rounded to 3 significant digits is -50.9.
(b) -50.893 rounded to 2 significant digits is -51.

28. **(a)** 7.004 rounded to 3 significant digits is 7.00.
(b) 7.004 rounded to 2 significant digits is 7.0.

29. **(a)** 5968 rounded to 3 significant digits is 5970.
(b) 5968 rounded to 2 significant digits is $6\bar{0}00$.

30. **(a)** 30.96 rounded to 3 significant digits is 31.0.
(b) 30.96 rounded to 2 significant digits is 31.

31. **(a)** 0.9449 rounded to 3 significant digits is 0.945.
(b) 0.9449 rounded to 2 significant digits is 0.94.

32. **(a)** 0.9999 rounded to 3 significant digits is 1.00.
(b) 0.9999 rounded to 2 significant digits is 1.0.

33. **(a)** Estimate: $13+1-2=12$
(b) Calculator: $12.78+1.0495-1.633=12.1965$, which is 12.20 to 0.01 precision

34. **(a)** Estimate: $4 \times 17 = 68$
(b) Calculator: $3.64(17.06) = 62.0984$, which is 62.1 to 3 significant digits

35. **(a)** Estimate $0.7 \times 4 - 9 = -6$
(b) Calculator: $0.6572 \times 3.94 - 8.651 = -6.061632$, which is -6.06 to 3 significant digits

36. **(a)** Estimate $40 - 26 \div 4 = 40 - 6.5 = 34$
(b) Calculator: $41.5 - 26.4 \div 3.7 = 34.3648649$, which is 34 to 2 significant digits

37. **(a)** Estimate $9 + (1)(4) = 9 + 4 = 13$
(b) Calculator: $8.75 + (1.2)(3.84) = 13.358$, which is 13 to 2 significant digits

38. **(a)** Estimate $30 - \dfrac{20}{2} = 30 - 10 = 20$
(b) Calculator: $28 - \dfrac{20.955}{2.2} = 18.475$, which is 18 to 2 significant digits

39. **(a)** Estimate $\dfrac{9(15)}{9+15} = \dfrac{135}{24} = 6$, to 1 significant digit
(b) Calculator: $\dfrac{8.75(15.32)}{8.75+15.32} = 5.569173$, which is 5.57 to 3 significant digits

40. **(a)** Estimate $\dfrac{9(4)}{2+5} = \dfrac{36}{7} = 5$, to 1 significant digit
(b) Calculator: $\dfrac{8.97(4.003)}{2.0+4.78} = 5.296$, which is 5.3 to 2 significant digits

41. **(a)** Estimate $4.5 - \dfrac{2(300)}{400} = 3.0$, to 2 significant digits
(b) Calculator: $4.52 - \dfrac{2.056(309.6)}{395.2} = 2.9093279$, which is 2.91 to 3 significant digits

42. **(a)** Estimate $8 + \dfrac{15}{2+2} = 12$, to 2 significant digits
(b) Calculator: $8.195 + \dfrac{14.9}{1.7+2.1} = 12.1160526$, which is 12 to 2 significant digits

43. $0.9788 + 14.9 = 15.8788$ since the least precise number in the question has 4 decimal places.

44. $17.311 - 22.98 = -5.669$ since the least precise number in the question has 3 decimal places.

45. $-3.142(65) = -204.23$, which is -204.2 because the least accurate number has 4 significant digits.

46. $8.62 \div 1728 = 0.004988$, which is 0.00499 because the least accurate number has 3 significant digits.

47. With a frequency listed as 2.75 MHz, the least possible frequency is 2.745 MHz, and the greatest possible frequency is 2.755 MHz. Any measurements between those limits would round to 2.75 MHz.

48. For an engine displacement stated at 2400 cm^3, the least possible displacement is 2350 cm^3, and the greatest possible displacement is 2450 cm^3. Any measurements between those limits would round to 2400 cm^3.

49. The speed of sound is $3.25 \text{ mi} \div 15 \text{ s} = 0.21666... \text{ mi/s} = 1144.0... \text{ ft/s}$. However, the least accurate measurement was time since it has only 2 significant digits. The correct answer is 1100 ft/s.

50. $4.4 \text{ s} - 2.72 \text{ s} = 1.68 \text{ s}$, but the answer must be given according to precision of the least precise measurement in the question, so the correct answer is 1.7 s.

51. **(a)** $2.2 + 3.8 \times 4.5 = 2.2 + (3.8 \times 4.5) = 19.3$

(b) $(2.2 + 3.8) \times 4.5 = 6.0 \times 4.5 = 27$

52. **(a)** $6.03 \div 2.25 + 1.77 = (6.03 \div 2.25) + 1.77 = 4.45$

(b) $6.03 \div (2.25 + 1.77) = 6.03 \div 4.02 = 1.5$

53. **(a)** $2 + 0 = 2$

(b) $2 - 0 = 2$

(c) $0 - 2 = -2$

(d) $2 \times 0 = 0$

(e) $2 \div 0 =$ error; from Section 1.2, an equation that has 0 in the denominator is undefined when the numerator is not also 0.

54. **(a)** $2 \div 0.0001 = 20\ 000$; $2 \div 0 =$ error

(b) $0.0001 \div 0.0001 = 1$; $0 \div 0 =$ error

(c) Any number divided by zero is undefined. Zero divided by zero is indeterminate.

55. $\pi = 3.14159265...$

(a) $\pi < 3.1416$

(b) $22 \div 7 = 3.1428$

$\pi < (22 \div 7)$

56. **(a)** $8 \div 33 = 0.2424... = 0.\overline{24}$

(b) $\pi = 3.14159265...$

57. **(a)** $1 \div 3 = 0.333...$ It is a rational number since it is a repeating decimal.

(b) $5 \div 11 = 0.454545...$ It is a rational number since it is a repeating decimal.

(c) $2 \div 5 = 0.400...$ It is a rational number since it is a repeating decimal (0 is the repeating part).

58. $124 \div 990 = 0.12525....$ the calculator may show the answer as 0.1252525253 because it has rounded up for the next 5 that doesn't fit on the screen.

59. $32.4 \text{ MJ} + 26.704 \text{ MJ} + 36.23 \text{ MJ} = 95.334 \text{ MJ}$. The answer must be to the same precision as the least precise measurement. The answer is 95.3 MJ.

60. We would compute $8(68.6) + 5(15.3) = 625.3$ and round to three significant digits for a total weight of 625 lb. The values 8 and 5 are exact.

61. We would compute $12(129) + 16(298.8) = 6328.8$ and round to three significant digits for a total weight of 6330 g. The values 12 and 16 are exact.

62. $V = (15.2\ \Omega + 5.64\ \Omega + 101.23\ \Omega) \times 3.55 \text{ A}$

$V = 122.07\ \Omega \times 3.55 \text{ A}$

$V = 433.3485 \text{ V}$

$V = 433 \text{ V}$ to 3 significant digits

63. $\dfrac{100(40.63+52.96)}{105.30+52.96} = 59.1386\ \% = 59.14\ \%$ to 4 signficiant digits

64. $T = \dfrac{50.45(9.80)}{1+100.9 \div 23} = 91.779$ N = 92 N to 2 significant digits

65. **(a)** Estimate $8 \times 5 - 10 = 30,$ to 1 significant digit.
(b) Calculator: $7.84 \times 4.932 - 11.317 = 27.34988$ which is 27.3 to 3 significant digits.

66. **(a)** Estimate $20 - 50 \div 10 = 15$ to 2 significant digit.
(b) Calculator: $21.6 - 53.14 \div 9.64 = 16.0875519$ which is 16.1 to 3 significant digits.

1.4 Exponents and Unit Conversions

1. $(-x^3)^2 = \left[(-1)x^3\right]^2 = (-1)^2(x^3)^2 = (1)x^6 = x^6$

2. $2x^0 = 2(1) = 2$

3. $x^3x^4 = x^{3+4} = x^7$

4. $y^2y^7 = y^{2+7} = y^9$

5. $2b^4b^2 = 2b^{4+2} = 2b^6$

6. $3k^5k = 3k^{5+1} = 3k^6$

7. $\dfrac{m^5}{m^3} = m^{5-3} = m^2$

8. $\dfrac{2x^6}{-x} = -2x^{6-1} = -2x^5$

9. $\dfrac{-n^5}{7n^9} = -\dfrac{n^{5-9}}{7} = -\dfrac{n^{-4}}{7} = -\dfrac{1}{7n^4}$

10. $\dfrac{3s}{s^4} = 3s^{1-4} = 3s^{-3} = \dfrac{3}{s^3}$

11. $\left(P^2\right)^4 = P^{2(4)} = P^8$

12. $\left(x^8\right)^3 = x^{8(3)} = x^{24}$

13. $\left(aT^2\right)^{30} = a^{30}T^{2(30)} = a^{30}T^{60}$

14. $\left(3r^2\right)^3 = (3)^3r^{2(3)} = 27r^6$

15. $\left(\frac{2}{b}\right)^3 = \frac{(2)^3}{b^3} = \frac{8}{b^3}$

16. $\left(\frac{F}{t}\right)^{20} = \frac{F^{20}}{t^{20}}$

17. $\left(\frac{x^2}{-2}\right)^4 = \frac{x^{2(4)}}{(-2)^4} = \frac{x^8}{16}$

18. $\left(\frac{3}{n^3}\right)^3 = \frac{(3)^3}{n^{3(3)}} = \frac{27}{n^9}$

19. $(8a)^0 = 1$

20. $-v^0 = -1$

21. $-3x^0 = -3(1) = -3$

22. $-(-2)^0 = -1(1) = -1$

23. $6^{-1} = \frac{1}{6^1} = \frac{1}{6}$

24. $-w^{-5} = -\frac{1}{w^5}$

25. $\frac{1}{R^{-2}} = R^2$

26. $\frac{1}{-t^{-48}} = -t^{48}$

27. $(-t^2)^7 = \left[(-1)(t^2)\right]^7 = (-1)^7 t^{2(7)} = (-1)t^{14} = -t^{14}$

28. $(-y^3)^5 = \left[(-1)(y^3)\right]^5 = (-1)^5 y^{3(5)} = (-1)y^{15} = -y^{15}$

29. $-\frac{L^{-3}}{L^{-5}} = -L^{-3-(-5)} = -L^2$

30. $2i^{40}i^{-70} = 2i^{40+(-70)} = 2i^{-30} = \frac{2}{i^{30}}$

31. $\frac{2v^4}{(2v)^4} = \frac{2v^4}{(2)^4(v^4)} = \frac{2v^4}{16v^4} = \frac{1}{8}$

32. $\dfrac{x^2x^3}{(x^2)^3} = \dfrac{x^{2+3}}{x^{2(3)}} = \dfrac{x^5}{x^6} = \dfrac{1}{x}$

33. $\dfrac{(n^2)^4}{(n^4)^2} = \dfrac{n^{2(4)}}{n^{4(2)}} = \dfrac{n^8}{n^8} = 1$

34. $\dfrac{(3t)^{-1}}{3t^{-1}} = \dfrac{(3)^{-1}t^{-1}}{3t^{-1}} = \dfrac{t}{3(3)t} = \dfrac{1}{9}$

35. $(\pi^0x^2a^{-1})^{-1} = \pi^{0(-1)}x^{2(-1)}a^{-1(-1)} = \pi^0x^{-2}a^1 = \dfrac{a}{x^2}$

36. $(3m^{-2}n^4)^{-2} = (3)^{-2}m^{-2(-2)}n^{4(-2)} = 3^{-2}m^4n^{-8} = \dfrac{m^4}{9n^8}$

37. $(-8g^{-1}s^3)^2 = (-8)^2g^{-1(2)}s^{3(2)} = \dfrac{64s^6}{g^2}$

38. $ax^{-2}(-a^2x)^3 = ax^{-2}(-1)^3(a^{2(3)})x^3 = -\dfrac{a(a^6)x^3}{x^2} = -a^{1+6}x^{3-2} = -a^7x$

39. $\left(\dfrac{4x^{-1}}{a^{-1}}\right)^{-3} = \dfrac{(4)^{-3}x^{-1(-3)}}{a^{-1(-3)}} = \dfrac{x^3}{64a^3}$

40. $\left(\dfrac{2b^2}{y^5}\right)^{-2} = \dfrac{(2)^{-2}b^{2(-2)}}{y^{5(-2)}} = \dfrac{b^{-4}}{4y^{-10}} = \dfrac{y^{10}}{4b^4}$

41. $\dfrac{15n^2T^5}{3n^{-1}T^6} = \dfrac{5n^{2-(-1)}}{T} = \dfrac{5n^3}{T}$

42. $\dfrac{(nRT^{-2})^{32}}{R^{-2}T^{32}} = \dfrac{n^{32}R^{32-(-2)}T^{-2(32)}}{T^{32}} = \dfrac{n^{32}R^{34}T^{-64}}{T^{32}} = \dfrac{n^{32}R^{34}}{T^{32-(-64)}} = \dfrac{n^{32}R^{34}}{T^{96}}$

43. $7(-4)-(-5)^2 = -28-25 = -53$

44. $6-|-2|^5-(-2)(8) = 6-32-(-16) = 6-32+16 = -10$

45. $-(-26.5)^2-(-9.85)^3 = -(702.25)-(-955.671625) = 253.421625$
which gets rounded to 253 because 702.25 and –955.671625 are both accurate to only 3 significant digits due to the original numbers having only 3 significant digits.

46. $-0.711^2-(-|-0.809|)^6 = (-1)(0.711)^2-(-0.809)^6 = (-1)(0.505521)-(0.2803439122) = -0.7858649122$
which gets rounded to 3 significant digits: –0.786.

47. $\dfrac{3.07(-|-1.86|)}{(-1.86)^4+1.596}=\dfrac{-5.7102}{11.96883216+1.596}=\dfrac{-5.7102}{13.56483216}=-0.420956185$
which gets rounded to 3 significant digits: –0.421.

48. $\dfrac{15.66^2-(-4.017)^4}{1.044(-3.68)}=\dfrac{245.2356-260.379822692}{-3.84192}=\dfrac{-15.144222692}{-3.84192}=3.941837074$
which gets rounded to 3 significant digits: 3.94.

49. $2.38(-60.7)^2-\dfrac{254}{1.17^3}=2.38(3684.49)-\dfrac{254}{1.601613}$
$=8769.0862-158.5901213339$
$=8610.4960786661$
which gets rounded to 3 significant digits: 8610.

50. $4.2(4.6)+\dfrac{0.889}{1.89-1.09^2}$
$=19.32+\dfrac{0.889}{1.89-1.1881}$
$=19.32+\dfrac{0.889}{0.7019}$
$=19.32+0.889880728$
$=20.209880728$
which gets rounded to 2 significant digits: $2\bar{0}$.

51. $\left(\dfrac{1}{x^{-1}}\right)^{-1}=\dfrac{1^{-1}}{x^{-1(-1)}}=\dfrac{1}{x}$, which is the reciprocal of x.

52. $\left(\dfrac{0.2-5^{-1}}{10^{-2}}\right)^0=\left(\dfrac{0.2-\frac{1}{5}}{\frac{1}{100}}\right)^0=\left(\dfrac{0}{0.01}\right)^0=0^0\neq 1$, since $a^0=1$ requires that $a\neq 0$.

53. If $a^3=5$, then
$a^{12}=a^{3(4)}$
$a^{12}=\left(a^3\right)^4$
$a^{12}=(5)^4$
$a^{12}=625$

54. For any negative value of a, a will be negative, and a^2 will be positive, making all values of $\dfrac{1}{a^2}$ greater than $\dfrac{1}{a}$.
Therefore, it is never the case for negative values of a, $a^{-2}<a^{-1}$.

55. $(x^a\cdot x^{-a})^5=(x^{a-a})^5=(x^0)^5=x^{0(5)}=x^0=1$, provided that $x\neq 0$.

56. $(-y^{a-b}\cdot y^{a+b})^2=((-1)y^{a-b+(a+b)})^2=(-1)^2(y^{2a})^2=y^{2a(2)}=y^{4a}$.

57. $$\left(\frac{kT}{hc}\right)^3 (GkThc)^2 c = \frac{k^3T^3}{h^3c^3} \cdot (G^2k^2T^2h^2c^2)c$$
$$= \frac{k^3T^3}{h^3c^3} \cdot (G^2k^2T^2h^2c^3)$$
$$= \frac{(G^2k^{2+3}T^{2+3}c^{3-3})}{h^1}$$
$$= \frac{G^2k^5T^5}{h}$$

58. $$GmM(mr)^{-1}(r^{-2}) = \frac{GmM}{mr^{1+2}} = \frac{GM}{r^3}$$

59. $$\pi\left(\frac{r}{2}\right)^3\left(\frac{4}{3\pi r^2}\right) = \pi\left(\frac{r^3}{8}\right)\left(\frac{4}{3\pi r^2}\right) = \frac{4r}{24} = \frac{r}{6}$$

60. $$\frac{gM(2\pi fM)^{-2}}{2\pi fC} = \frac{gM}{2\pi fC(2\pi fM)^2}$$
$$= \frac{gM}{2\pi fC(4\pi^2 f^2 M^2)}$$
$$= \frac{gM}{8\pi^3 f^3 CM^2}$$
$$= \frac{g}{8\pi^3 f^3 CM}$$

61. $$2500\left(1+\frac{0.042}{4}\right)^{24} = \$2500(1.0105)^{24}$$
$$= \$2500(1.28490602753)$$
$$= \$3212.26700688$$
$$= \$3212.27$$

62. $$\frac{6.85(1000-20(6.85)^2+(6.85)^3)}{1850} = \frac{6.85(1000-20(46.9225)+321.419125)}{1850}$$
$$= \frac{6.85(1321.419125-938.45)}{1850}$$
$$= \frac{6.85(382.969125)}{1850}$$
$$= \frac{2623.33850625}{1850}$$
$$= 1.418020814$$
$$= 1.42 \text{ cm}$$

63. If $f(n) = \left(1+\frac{1}{n}\right)^n$ then $f(1) = 2^1 = 2.000$, $f(10) = 1.1^{10} = 2.594$, $f(100) = 1.01^{100} = 2.705$, and $f(1000) = 1.001^{1000} = 2.717$.

64. We have 1 TB $= 2^{10}$ GB $= 2^{10}(2^{10}$ MB$) = 2^{10}(2^{10}(2^{20}$ bytes$)) = 2^{10+10+20}$ bytes $= 2^{40}$ bytes

65. $\left(28.2\dfrac{\text{ft}}{\text{s}}\right)(9.81\text{ s}) = 276.642$ ft which is rounded to 277 ft.

66. $\left(40.5\dfrac{\text{mi}}{\text{gal}}\right)(3.7\text{ gal}) = 149.85$ mi which is rounded to 150 mi.

67. $\left(7.25\dfrac{\text{m}}{\text{s}^2}\right)\left(\dfrac{1\text{ ft}}{0.3048\text{ m}}\right)\left(\dfrac{60\text{ s}}{1\text{ min}}\right)^2 = 85{,}629.92\ \dfrac{\text{ft}}{\text{min}^2}$ which is rounded to 85,600 $\dfrac{\text{ft}}{\text{min}^2}$.

68. $\left(238\dfrac{\text{kg}}{\text{m}^3}\right)\left(\dfrac{1000\text{ g}}{1\text{ kg}}\right)\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)^3 = 0.238\ \dfrac{\text{g}}{\text{cm}^3}$.

69. $15.7\text{ qt} = 15.7\text{ qt}\times\left(\dfrac{1\text{ L}}{1.057\text{ qt}}\right) = 14.8533586$ L which is rounded to 14.9 L.

70. $7.50\text{ W} = 7.50\text{ W}\times\left(\dfrac{1\text{ hp}}{746.0\text{ W}}\right) = 0.01005362$ hp which is rounded to 0.0101 hp.

71. $245\text{ cm}^2 = 245\text{ cm}^2\times\left(\dfrac{1\text{ in}}{2.54\text{ cm}}\right)^2 = 37.975076\text{ in}^2$ which is rounded to 38.0 in^2.

72. $85.7\text{ mi}^2 = 85.7\text{ mi}^2\times\left(\dfrac{1\text{ km}}{0.6214\text{ mi}}\right)^2 = 221.941401\text{ km}^2$ which is rounded to 222 km^2.

73. $65.2\dfrac{\text{m}}{\text{s}} = 65.2\dfrac{\text{m}}{\text{s}}\times\left(\dfrac{60\text{ s}}{1\text{ min}}\right)\times\left(\dfrac{1\text{ ft}}{0.3048\text{ m}}\right) = 12834.6457\ \dfrac{\text{ft}}{\text{min}}$ which is rounded to 12800 $\dfrac{\text{ft}}{\text{min}}$.

74. $25.0\dfrac{\text{mi}}{\text{gal}} = 25.0\dfrac{\text{mi}}{\text{gal}}\times\left(\dfrac{1\text{ km}}{0.6214\text{ mi}}\right)\times\left(\dfrac{1\text{ gal}}{3.785\text{ L}}\right) = 10.6292562\ \dfrac{\text{km}}{\text{L}}$ which is rounded to 10.6 $\dfrac{\text{km}}{\text{L}}$.

75. $15.6\text{ in} = 15.6\text{ in}\times\left(\dfrac{2.54\text{ cm}}{1\text{ in}}\right) = 39.624$ cm which is rounded to 39.6 cm.

76. $12{,}500\text{ mi} = 12{,}500\text{ mi}\times\left(\dfrac{1\text{ km}}{0.6214\text{ mi}}\right) = 20{,}115.8674$ km which is rounded to 20,100 km.

77. $575{,}000\dfrac{\text{gal}}{\text{day}} = 575{,}000\dfrac{\text{gal}}{\text{day}}\times\left(\dfrac{1\text{ day}}{24\text{ hr}}\right)\times\left(\dfrac{3.785\text{ L}}{1\text{ gal}}\right) = 90{,}682.2917\ \dfrac{\text{L}}{\text{hr}}$ which is rounded to 90,700 $\dfrac{\text{L}}{\text{hr}}$.

78. $85\dfrac{\text{gal}}{\text{min}} = 85\dfrac{\text{gal}}{\text{min}}\times\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\times\left(\dfrac{3.785\text{ L}}{1\text{ gal}}\right) = 5.3620833\ \dfrac{\text{L}}{\text{s}}$ which is rounded to 5.4 $\dfrac{\text{L}}{\text{s}}$.

79. $1130\dfrac{\text{ft}}{\text{s}} = 1130\dfrac{\text{ft}}{\text{s}}\times\left(\dfrac{60\text{ s}}{1\text{ min}}\right)\times\left(\dfrac{60\text{ min}}{1\text{ hr}}\right)\times\left(\dfrac{0.3084\text{ m}}{1\text{ ft}}\right)\times\left(\dfrac{1\text{ km}}{1000\text{ m}}\right) = 1254.5712\ \dfrac{\text{km}}{\text{hr}}$ which is rounded to 1250 $\dfrac{\text{km}}{\text{hr}}$.

80. $7200\frac{\text{km}}{\text{hr}} = 7200\frac{\text{km}}{\text{hr}}\times\left(\frac{1\text{ hr}}{60\text{ min}}\right)\times\left(\frac{1\text{ min}}{60\text{ s}}\right)\times\left(\frac{1000\text{ m}}{1\text{ km}}\right) = 2000\ \frac{\text{m}}{\text{s}}.$

81. $14.7\frac{\text{lb}}{\text{in}^2} = 14.7\frac{\text{lb}}{\text{in}^2}\times\left(\frac{4.448\text{ N}}{1\text{ lb}}\right)\times\left(\frac{1\text{ in}}{2.54\text{ cm}}\right)^2\times\left(\frac{100\text{ cm}}{1\text{ m}}\right)^2 = 101,347.883\ \frac{\text{N}}{\text{m}^2}$ which is rounded to 101,000 Pa.

82. $62.4\frac{\text{lb}}{\text{ft}^3} = 62.4\frac{\text{lb}}{\text{ft}^3}\times\ \frac{1\text{ kg}}{2.205\text{ lb}}\ \times\ \frac{1\text{ ft}}{0.3048\text{ m}}^{3} = 999.381\ \frac{\text{kg}}{\text{m}^3}$ which is rounded to 999 $\frac{\text{kg}}{\text{m}^3}$.

(The actual value is 1000 $\frac{\text{kg}}{\text{m}^3}$.)

1.5 Scientific Notation

1. $8.06\times10^3 = 8060$

2. $750\ 000\ 000\ 000^{-1} = (7.5\times10^{11})^{-1}$

$= 7.5^{-1}\times10^{-11}$

$= 0.1333...\times10^{-11}$

$= 1.33\times10^{-12}$

rounded to 3 significant digits.

3. $4.5\times10^4 = 45{,}000$

4. $6.8\times10^7 = 68{,}000{,}000$

5. $2.01\times10^{-3} = 0.00201$

6. $9.61\times10^{-5} = 0.0000961$

7. $3.23\times10^0 = 3.23\times1 = 3.23$

8. $8\times10^0 = 8\times1 = 8$

9. $1.86\times10 = 18.6$

10. $1\times10^{-1} = 0.1$

11. $4000 = 4\times10^3$

12. $56\ 000 = 5.6\times10^4$

13. $0.0087 = 8.7\times10^{-3}$

14. $0.00074 = 7.4\times10^{-4}$

15. $609,000,000 = 6.09\times10^8$

16. $10 = 1\times10^{1}$

17. $0.0528 = 5.28\times10^{-2}$

18. $0.0000908 = 9.08\times10^{-5}$

19. $28{,}000(2{,}000{,}000{,}000) = 2.8\times10^{4}(2\times10^{9}) = 5.6\times10^{13}$

20. $50{,}000(0.006) = 5\times10^{4}(6\times10^{-3}) = 300 = 3\times10^{2}$

21. $\dfrac{88{,}000}{0.0004} = \dfrac{8.8\times10^{4}}{4\times10^{-4}} = 2.2\times10^{8}$

22. $\dfrac{0.00003}{6{,}000{,}000} = \dfrac{3\times10^{-5}}{6\times10^{6}} = 5\times10^{-12}$

23. $35{,}600{,}000 = 35.6\times10^{6}$

24. $0.0000056 = 5.6\times10^{-6}$

25. $0.0973 = 97.3\times10^{-3}$

26. $925{,}000{,}000{,}000 = 925\times10^{9}$

27. $0.000000475 = 475\times10^{-9}$

28. $370{,}000 = 370\times10^{3}$

29. $2\times10^{-35} + 3\times10^{-34} = 0.2\times10^{-34} + 3\times10^{-34} = 3.2\times10^{-34}$

30. $5.3\times10^{12} - 3.7\times10^{10} = 530\times10^{10} - 3.7\times10^{10} = 526.3\times10^{10} = 5.263\times10^{12}$

31. $(1.2\times10^{29})^{3} = 1.2^{3}\times10^{29(3)} = 1.728\times10^{87}$

32. $(2\times10^{-16})^{-5} = 2^{-5}\times10^{-16(-5)} = 0.031\,25\times10^{80} = 3.125\times10^{78}$

33. $1320(649{,}000)(85.3) = 7.3074804\times10^{10}$
which gets rounded to 7.31×10^{10}.

34. $0.0000569(3{,}190{,}000) = 181.511$
which gets rounded to 1.82×10^{2}.

35. $\dfrac{0.0732(6710)}{0.00134(0.0231)} = \dfrac{491.172}{0.000030954} = 1.5867803\times10^{7}$
which gets rounded to 1.59×10^{7}.

36. $\frac{0.00452}{2430(97,100)} = \frac{0.00452}{235,953,000} = 1.915635741\times10^{-11}$
which gets rounded to 1.92×10^{-11}.

37. $(3.642\times10^{-8})(2.736\times10^{5}) = 9.964512\times10^{-3}$
which gets rounded to 9.965×10^{-3}.

38. $\frac{(7.309\times10^{-1})^2}{5.9843(2.5036\times10^{-20})} = \frac{0.53421481}{1.49787029\times10^{-19}} = 3.56656723394\times10^{18}$
which gets rounded to 3.567×10^{18}.

39. $\frac{(3.69\times10^{-7})(4.61\times10^{21})}{0.0504} = \frac{1.70109\times10^{15}}{0.0504} = 3.37517857142\times10^{16}$
which gets rounded to 3.38×10^{16}.

40. $\frac{(9.907\times10^{7})(1.08\times10^{12})^2}{(3.603\times10^{-5})(2054)} = \frac{(9.907\times10^{7})(1.1664\times10^{24})}{0.07400562} = \frac{1.01555248\times10^{32}}{0.07400562} = 1.56143882045\times10^{33}$
which gets rounded to 1.56×10^{33}.

41. 500,000,000 tweets $= 5\times10^{8}$ tweets

42. 17,200,000,000 bytes $= 1.72\times10^{10}$ bytes

43. 0.000003 W $= 3\times10^{-6}$ W

44. 0.0075 mm $= 7.5\times10^{-3}$ mm

45. 1,200,000,000 Hz = 1.2×10^{9} Hz

46. $1.84\times10^{12} = 1,840,000,000,000$

47. $12,000,000,000 \text{ m}^2 = 1.2\times10^{10} \text{ m}^2$

48. 3.086×10^{16} m = 30,860,000,000,000,000 m

49. 1.6×10^{-12} W = 0.0000000000016 W

50. $2.4\times10^{-43} = 0.00000000000000000000000000000000000000000024$

51. **(a)** $2300 = 2.3\times10^{3} = 2.3$ kW
(b) $0.23 = 230\times10^{-3} = 230$ mW
(c) $2,300,000 = 2.3\times10^{6} = 2.3$ MW
(d) $0.00023 = 230\times10^{-6} = 230\ \mu$W

52. **(a)** $8090000 = 8.09\times10^{6} = 8.09$ MΩ
(b) $809\ 000 = 809\times10^{3} = 809$ kΩ
(c) $0.0809 = 80.9\times10^{-3} = 80.9$ mΩ

53. **(a)** $\text{googol} = 1\times10^{100} = 10^{100}$

(b) $\text{googolplex} = 10^{\text{googol}} = 10^{10^{100}}$

54. $\text{googol} = 10^{100}$, so to find the ratio $\frac{10^{100}}{10^{79}} = 10^{100-79} = 10^{21}$

A googol is 10^{21} times larger than the number of electrons in the universe.

55. $\text{earth's diameter} = \frac{\text{sun's diameter}}{110} = \frac{1.4\times10^{9}\text{ m}}{110} = 1.27272\times10^{7}$ m which is rounded to 1.3×10^{7} m.

56. $2^{30} = 1{,}073{,}741{,}824 = 1.073741824\times10^{9} \approx 1\times10^{9}$

57. $\frac{7.5\times10^{-15}\text{s}}{\text{addition}}\times5.6\times10^{6}\text{additions} = 4.2\times10^{-8}\text{s}$

58. $0.000000039\ \% = 0.00000000039$

$0.00000000039\times0.085\text{ mg} = 3.315\times10^{-11}\text{mg} = 3.3\times10^{-11}\text{mg}$

59. $0.078\text{ s}\times2.998\times10^{8}\frac{\text{m}}{\text{s}} = 2.3384400\times10^{7}$ m which rounds to 2.3×10^{7} m

60. **(a)** $1\text{ day}\times\frac{24\text{ h}}{\text{day}}\times\frac{60\text{ min}}{\text{h}}\times\frac{60\text{ s}}{\text{min}} = 86400\text{ s} = 8.64\times10^{4}\text{ s}$

(b) $100\text{ year}\times\frac{365.25\text{ day}}{\text{year}}\times\frac{24\text{ h}}{\text{day}}\times\frac{60\text{ min}}{\text{h}}\times\frac{60\text{ s}}{\text{min}} = 3\ 155\ 760\ 000\text{ s} = 3.155\ 760\ 0\times10^{9}\text{ s}$

61. $\frac{1.66\times10^{-27}\text{ kg}}{\text{amu}}\times\frac{1.6\times10^{1}\text{ amu}}{\text{oxygen atoms}}\times1.25\times10^{8}\text{ oxygen atoms} = 3.32\times10^{-18}\text{ kg}$

62. $W = kT^{4}$

$W = 5.7\times10^{-8}\text{ W/K}^{4}\times(3.03\times10^{2}\text{ K})^{4}$

$W = 5.7\times10^{-8}\text{ W/K}^{4}\times8.428892481\times10^{9}\text{ K}^{4}$

$W = 4.80446871417\times10^{2}\text{ W}$

$W = 4.8\times10^{2}\text{ W}$

63. $R = \frac{k}{d^{2}} = \frac{2.196\times10^{-8}\ \Omega\cdot\text{m}^{2}}{\left(7.998\times10^{-5}\text{m}\right)^{2}} = \frac{2.196\times10^{-8}\ \Omega\cdot\text{m}^{2}}{6.396\ 800\ 4\times10^{-9}\text{m}^{2}} = 3.432\ 966\ 268\ 57\ \Omega = 3.433\ \Omega$

64. $\frac{1.496\times10^{8}\text{km}}{\text{AU}}\times\frac{\text{AU}}{4.99\times10^{2}\text{s}} = 2.99799599198\times10^{5}\text{ km/s} = 2.998\times10^{5}\text{ km/s}$

This is the same speed mentioned in Question 56 as the speed of radio waves.

1.6 Roots and Radicals

1. $-\sqrt[3]{64} = -\sqrt[3]{(4)^3} = -4$

2. $\sqrt{(15)(5)}$
Neither 15 nor 5 is a perfect square, so this expression is not as useful. However, if we further factor the 15 to $\sqrt{(3)(5)(5)} = \sqrt{3(5)^2} = 5\sqrt{3}$, the result can still be obtained.

3. $\sqrt{16\times 9} = \sqrt{144} = \sqrt{12^2} = 12$

4. $-\sqrt{-64}$ is still imaginary because an even root (in this case $n = 2$) of a negative number is imaginary, regardless of the numerical factor placed in front of the root.

5. $\sqrt{49} = \sqrt{7^2} = 7$

6. $\sqrt{225} = \sqrt{(25)(9)} = \sqrt{25}\times\sqrt{9} = 5\times 3 = 15$

7. $-\sqrt{121} = -\sqrt{11^2} = -11$

8. $-\sqrt{36} = -\sqrt{6^2} = -6$

9. $-\sqrt{64} = -\sqrt{8^2} = -8$

10. $\sqrt{0.25} = \sqrt{\dfrac{1}{4}} = \dfrac{\sqrt{1}}{\sqrt{4}} = \dfrac{1}{2} = 0.5$

11. $\sqrt{0.09} = \sqrt{\dfrac{9}{100}} = \dfrac{\sqrt{9}}{\sqrt{100}} = \dfrac{3}{10} = 0.3$

12. $-\sqrt{900} = -\sqrt{(9)(100)} = -\sqrt{9}\times\sqrt{100} = -3\times 10 = -30$

13. $\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

14. $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

15. $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

16. $-\sqrt[5]{-32} = -\sqrt[5]{(-2)^5} = -(-2) = 2$

17. $\left(\sqrt{5}\right)^2 = \sqrt{5}\times\sqrt{5} = 5$

18. $\left(\sqrt[3]{31}\right)^3 = \sqrt[3]{31}\times\sqrt[3]{31}\times\sqrt[3]{31} = 31$

19. $\left(-\sqrt[3]{-47}\right)^3=\left(-1\right)^3\left(\sqrt[3]{-47}\right)^3=(-1)(-47)=47$

20. $\left(\sqrt[5]{-23}\right)^5=-23$

21. $\left(-\sqrt[4]{53}\right)^4=\left(-1\right)^4\left(\sqrt[4]{53}\right)^4=(1)(53)=53$

22. $\sqrt{75}=\sqrt{(25)(3)}=\sqrt{25}\times\sqrt{3}=5\sqrt{3}$

23. $\sqrt{18}=\sqrt{(9)(2)}=\sqrt{9}\times\sqrt{2}=3\sqrt{2}$

24. $-\sqrt{32}=-\sqrt{(16)(2)}=-\sqrt{16}\times\sqrt{2}=-4\sqrt{2}$

25. $\sqrt{1200}=\sqrt{(100)(4)(3)}=\sqrt{100}\times\sqrt{4}\times\sqrt{3}=10\times2\times\sqrt{3}=20\sqrt{3}$

26. $\sqrt{50}=\sqrt{(25)(2)}=\sqrt{25}\times\sqrt{2}=5\times\sqrt{2}=5\sqrt{2}$

27. $2\sqrt{84}=2\sqrt{(4)(21)}=2\times\sqrt{4}\times\sqrt{21}=2\times2\times\sqrt{21}=4\sqrt{21}$

28. $\dfrac{\sqrt{108}}{2}=\dfrac{\sqrt{(36)(3)}}{2}=\dfrac{\sqrt{36}\times\sqrt{3}}{2}=\dfrac{6\times\sqrt{3}}{2}=3\sqrt{3}$

29. $\sqrt{\dfrac{80}{|3-7|}}=\sqrt{\dfrac{80}{4}}=\sqrt{20}=\sqrt{4\times5}=\sqrt{4}\times\sqrt{5}=2\times\sqrt{5}=2\sqrt{5}$

30. $\sqrt{81\times10^2}=\sqrt{81}\times\sqrt{10^2}=9\times10=90$

31. $\sqrt[3]{-8^2}=\sqrt[3]{-64}=\sqrt[3]{(-4)^3}=-4$

32. $\sqrt[4]{9^2}=\sqrt[4]{81}=\sqrt[4]{3^4}=3$

33. $\dfrac{7^2\sqrt{81}}{(-3)^2\sqrt{49}}=\dfrac{(49)(9)}{(9)(7)}=\dfrac{(49)\cancel{(9)}}{\cancel{(9)}(7)}=7$

34. $\dfrac{2^5\sqrt[5]{243}}{-3\sqrt{144}}=-\dfrac{32\sqrt[5]{3^5}}{3\sqrt{12^2}}=-\dfrac{(32)\cancel{(3)}}{\cancel{(3)}(12)}=-\dfrac{8}{3}$

35. $\sqrt{36+64}=\sqrt{100}=\sqrt{10^2}=10$

36. $\sqrt{25+144}=\sqrt{169}=\sqrt{13^2}=13$

37. $\sqrt{3^2+9^2}=\sqrt{9+81}=\sqrt{90}=\sqrt{(9)(10)}=\sqrt{9}\times\sqrt{10}=3\sqrt{10}$

38. $\sqrt{8^2-4^2}=\sqrt{64-16}=\sqrt{48}=\sqrt{(16)(3)}=\sqrt{16}\times\sqrt{3}=4\sqrt{3}$

39. $\sqrt{85.4}=9.24121204171$,which is rounded to 9.24

40. $\sqrt{3762}=61.3351449007$,which is rounded to 61.34

41. $\sqrt{0.8152}=0.9028842672$,which is rounded to 0.9029

42. $\sqrt{0.0627}=0.25039968051$,which is rounded to 0.250

43. **(a)** $\sqrt{1296+2304}=\sqrt{3600}=60$, which is expressed as 60.00
(b) $\sqrt{1296}+\sqrt{2304}=36+48=84$, which is expressed as 84.00

44. **(a)** $\sqrt{10.6276+2.1609}=\sqrt{12.7885}=3.57610122899$,which is rounded to 3.57610
(b) $\sqrt{10.6276}+\sqrt{2.1609}=3.26+1.47=4.73$,which is expressed as 4.7300

45. **(a)**
$$\begin{aligned}\sqrt{0.0429^2-0.0183^2}&=\sqrt{0.00184041-0.00033489}\\&=\sqrt{0.00150552}\\&=0.03880103091\\&=0.0388\end{aligned}$$
(b)
$$\begin{aligned}\sqrt{0.0429^2}-\sqrt{0.0183^2}&=0.0429-0.0183\\&=0.0246\end{aligned}$$

46. **(a)**
$$\begin{aligned}\sqrt{3.625^2+0.614^2}&=\sqrt{13.140625+0.376996}\\&=\sqrt{13.517621}\\&=3.67663174658\\&=3.677\end{aligned}$$
(b)
$$\begin{aligned}\sqrt{3.625^2}+\sqrt{0.614^2}&=3.625+0.614\\&=4.239\end{aligned}$$

47. $\sqrt{24s}=\sqrt{(24)(150)}=\sqrt{3600}=60$ mi/h

48.
$$\begin{aligned}\sqrt{Z^2-X^2}&=\sqrt{(5.362\ \Omega)^2-(2.875\ \Omega)^2}\\&=\sqrt{28.751044\ \Omega^2-8.265625\ \Omega^2}\\&=\sqrt{20.485419\ \Omega^2}\\&=4.52608208056\ \Omega\\&=4.526\ \Omega\end{aligned}$$

49. $\sqrt{\dfrac{B}{d}} = \sqrt{\dfrac{2.18\times10^9\text{ Pa}}{1.03\times10^3\text{ kg/m}^3}}$

$= \sqrt{2116504.85436\ \dfrac{\text{N/m}^2}{\text{kg/m}^3}}$

$= \sqrt{2116504.85436\ \dfrac{\left(\text{kg}\cdot\text{m/s}^2\right)/\text{m}^2}{\text{kg/m}^3}}$

$= \sqrt{2116504.85436\text{ m}^2/\text{s}^2}$

$= 1454.82124481\text{ m/s}$

$= 1450\text{ m/s}$

50. $\sqrt{40m} = \sqrt{(40)(75)}$

$= \sqrt{3000}$

$= 54.7722557505$

$= 55\text{ m/s}$

51. $\sqrt{w^2+h^2} = \sqrt{(52.3\text{ in})^2+(29.3\text{ in})^2}$

$= \sqrt{2735.29\text{ in}^2+858.49\text{ in}^2}$

$= \sqrt{3593.78\text{ in}^2}$

$= 59.948144258\text{ in}$

$= 59.9\text{ in}$

52. $100\left(1-\sqrt{\dfrac{V}{C}}\right) = 100\left(1-\sqrt{\dfrac{24000}{38000}}\right)$

$= 100\left(1-\sqrt{0.63157895}\right)$

$= 100(1-0.79471941589)$

$= 100(0.2052805841)$

$= 20.52805841\ \%$

$= 21\ \%$

53. $\sqrt{gd} = \sqrt{(9.8)(3500)}$

$= \sqrt{34300}$

$= 185.20259$

$= 190\text{ m/s}$

54. $\sqrt{1.27\times10^4 h+h^2} = \sqrt{1.27\times10^4(9500)+(9500)^2}$

$= \sqrt{1.2065\times10^8+9.025\times10^7}$

$= \sqrt{2.109\times10^8}$

$= 14522.3965$

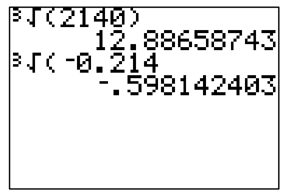
which is rounded to 15000 km

55. $\sqrt{a^2} = a$ is not necessarily true for negative values of *a* because a^2 will be a positive number, regardless whether *a* is negative or positive.The principal root calculated is assumed to be positive, but there are always two solutions to a square root, $\sqrt{a^2} = \pm a$ since $(+a)^2 = a^2$ and $(-a)^2 = a^2$ (see the introduction to this chapter section),so it is sometimes true and sometimes false for negative values of *a*, depending on which root solution is desired. If *only principal roots* are considered, then it will *not* be true for negative values of *a*. For example, $\sqrt{(-4)^2} = \sqrt{16} = 4 \neq -4$.

56. **(a)** $x > \sqrt{x}$ when $x > 1$. Any number greater than 1 will have a square root that is smaller than itself. For example, $2 > \sqrt{2} = 1.41$

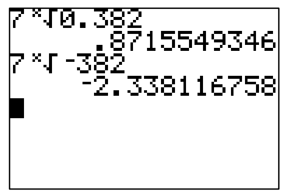
(b) $x = \sqrt{x}$ when $x = 1$ or $x = 0$ because the only numbers that are their own squares are 0 and 1 (i.e., $0^2 = 0$ and $1^2 = 1$).

(c) $x < \sqrt{x}$ when $0 < x < 1$. Any number between 0 and 1 will have a square root larger than itself. For example, $0.25 < \sqrt{0.25} = 0.5$



57. **(a)** $\sqrt[3]{2140} = 12.8865874254$,which is rounded to 12.9

(b) $\sqrt[3]{-0.214} = -0.59814240297$,which is rounded to –0.598



58. **(a)** $\sqrt[7]{0.382} = 0.87155493458$,which is rounded to 0.872

(b) $\sqrt[7]{-382} = -2.33811675837$,which is rounded to –2.34

59. $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2(3.1416)\sqrt{0.250(40.52\times10^{-6})}}$

$= \dfrac{1}{6.2832\sqrt{10.0625\times10^{-6}}}$

$= \dfrac{1}{6.2832(0.003172144385)}$

$= \dfrac{1}{0.0199312175998}$

$= 50.172549$

which is rounded to 50.2 Hz

60. standard deviation = $\sqrt{\text{variance}}$

$= \sqrt{80.5\text{ kg}^2}$

$= 8.972179222$ kg

which is rounded to 8.97 kg

1.7 Addition and Subtraction of Algebraic Expressions

1. $3x + 2y - 5y = 3x - 3y$

2. $3c - (2b - c) = 3c - 2b + c = -2b + 4c$

3. $$\begin{aligned}3ax-[(ax-5s)-2ax] &= 3ax-[ax-5s-2ax]\\ &= 3ax-[-ax-5s]\\ &= 3ax+ax+5s\\ &= 4ax+5s\end{aligned}$$

4. $$\begin{aligned}3a^2b-\{a-[2a^2b-(a+2b)]\} &= 3a^2b-\{a-[2a^2b-a-2b]\}\\ &= 3a^2b-\{a-2a^2b+a+2b\}\\ &= 3a^2b-\{2a-2a^2b+2b\}\\ &= 3a^2b-2a+2a^2b-2b\\ &= 5a^2b-2a-2b\end{aligned}$$

5. $5x+7x-4x=8x$

6. $6t-3t-4t=-t$

7. $2y-y+4x=y+4x$

8. $-4C+L-6C=-10C+L$

9. $3t-4s-3t-s=0t-5s=-5s$

10. $-8a-b+12a+b=4a+0b=4a$

11. $2F-2T-2+3F-T=5F-3T-2$

12. $x-2y-3x-y+z=-2x-3y+z$

13. $a^2b-a^2b^2-2a^2b=-a^2b-a^2b^2$

14. $-xy^2-3x^2y^2+2xy^2=xy^2-3x^2y^2$

15. $2p+(p-6-2p)=2p-6-p=p-6$

16. $5+(3-4n+p)=5+3-4n+p=-4n+p+8$

17. $v-(7-9x+2v)=v-7+9x-2v=-v+9x-7$

18. $-2a-\frac{1}{2}(b-a)=-2a-\frac{1}{2}b+\frac{1}{2}a=-\frac{3}{2}a-\frac{1}{2}b$

19. $2-3-(4-5a)=-1-4+5a=5a-5$

20. $\sqrt{A}+(h-2\sqrt{A})-3\sqrt{A}=\sqrt{A}+h-2\sqrt{A}-3\sqrt{A}=-4\sqrt{A}+h$

21. $(a-3)+(5-6a)=a-3+5-6a=-5a+2$

22. $(4x-y)-(-2x-4y)=4x-y+2x+4y=6x+3y$

23. $-(t-2u)+(3u-t)=-t+2u+3u-t=-2t+5u$

24. $-2(6x-3y)-(5y-4x)=-12x+6y-5y+4x=-8x+y$

25. $3(2r+s)-(-5s-r)=6r+3s+5s+r=7r+8s$

26. $3(a-b)-2(a-2b)=3a-3b-2a+4b=a+b$

27. $-7(6-3j)-2(j+4)=-42+21j-2j-8=19j-50$

28. $-(5t+a^2)-2(3a^2-2st)=-5t-a^2-6a^2+4st=-7a^2+4st-5t$

29.
$$\begin{aligned}-[(4-6n)-(n-3)]&=-[4-6n-n+3]\\&=-[-7n+7]\\&=7n-7\end{aligned}$$

30.
$$\begin{aligned}-[(A-B)-(B-A)]&=-[A-B-B+A]\\&=-[2A-2B]\\&=-2A+2B\end{aligned}$$

31.
$$\begin{aligned}2[4-(t^2-5)]&=2[4-t^2+5]\\&=2[-t^2+9]\\&=-2t^2+18\end{aligned}$$

32.
$$\begin{aligned}-3[-3-\frac{2}{3}(-a-4)]&=-3[-3+\frac{2}{3}a+\frac{8}{3}]\\&=-3[\frac{2}{3}a-\frac{1}{3}]\\&=-2a+1\end{aligned}$$

33.
$$\begin{aligned}-2[-x-2a-(a-x)]&=-2[-x-2a-a+x]\\&=-2[-3a]\\&=6a\end{aligned}$$

34.
$$\begin{aligned}-2[-3(x-2y)+4y]&=-2[-3x+6y+4y]\\&=-2[-3x+10y]\\&=6x-20y\end{aligned}$$

35.
$$\begin{aligned}aZ-[3-(aZ+4)]&=aZ-[3-aZ-4]\\&=aZ-[-aZ-1]\\&=aZ+aZ+1\\&=2aZ+1\end{aligned}$$

36.
$$\begin{aligned}9v-[6-(-v-4)+4v]&=9v-[6+v+4+4v]\\&=9v-[5v+10]\\&=9v-5v-10\\&=4v-10\end{aligned}$$

37. $$\begin{aligned} 5z-\{8-[4-(2z+1)]\} &= 5z-\{8-[4-2z-1]\} \\ &= 5z-\{8-4+2z+1\} \\ &= 5z-\{5+2z\} \\ &= 5z-5-2z \\ &= 3z-5 \end{aligned}$$

38. $$\begin{aligned} 7y-\{y-[2y-(x-y)]\} &= 7y-\{y-[2y-x+y]\} \\ &= 7y-\{y-[3y-x]\} \\ &= 7y-\{y-3y+x\} \\ &= 7y-\{-2y+x\} \\ &= 7y+2y-x \\ &= -x+9y \end{aligned}$$

39. $$\begin{aligned} 5p-(q-2p)-[3q-(p-q)] &= 5p-q+2p-[3q-p+q] \\ &= 5p-q+2p-[4q-p] \\ &= 7p-q-4q+p \\ &= 8p-5q \end{aligned}$$

40. $$\begin{aligned} -(4-\sqrt{LC})-[(5\sqrt{LC}-7)-(6\sqrt{LC}+2)] &= -4+\sqrt{LC}-[5\sqrt{LC}-7-6\sqrt{LC}-2] \\ &= -4+\sqrt{LC}-[-\sqrt{LC}-9] \\ &= -4+\sqrt{LC}+\sqrt{LC}+9 \\ &= 2\sqrt{LC}+5 \end{aligned}$$

41. $$\begin{aligned} -2\{-(4-x^2)-[3+(4-x^2)]\} &= -2\{-4+x^2-[3+4-x^2]\} \\ &= -2\{-4+x^2-3-4+x^2\} \\ &= -2\{2x^2-11\} \\ &= -4x^2+22 \end{aligned}$$

42. $$\begin{aligned} -\{-[-(x-2a)-b]-(a-x)\} &= -\{-[-x+2a-b]-a+x\} \\ &= -\{x-2a+b-a+x\} \\ &= -\{-3a+b+2x\} \\ &= 3a-b-2x \end{aligned}$$

43. $$\begin{aligned} 5V^2-(6-(2V^2+3)) &= 5V^2-(6-2V^2-3) \\ &= 5V^2-(-2V^2+3) \\ &= 5V^2+2V^2-3 \\ &= 7V^2-3 \end{aligned}$$

44. $$\begin{aligned} -2F+2((2F-1)-5) &= -2F+2(2F-1-5) \\ &= -2F+2(2F-6) \\ &= -2F+4F-12 \\ &= 2F-12 \end{aligned}$$

45. $-(3t-(7+2t-(5t-6))) = -(3t-(7+2t-5t+6))$
$= -(3t-(-3t+13))$
$= -(3t+3t-13)$
$= -(6t-13)$
$= -6t+13$

46. $a^2-2(x-5-(7-2(a^2-2x)-3x)) = a^2-2(x-5-(7-2a^2+4x-3x))$
$= a^2-2(x-5-(7-2a^2+x))$
$= a^2-2(x-5-7+2a^2-x)$
$= a^2-2(2a^2-12)$
$= a^2-4a^2+24$
$= -3a^2+24$

47. $-4[4R-2.5(Z-2R)-1.5(2R-Z)] = -4[4R-2.5Z+5R-3R+1.5Z]$
$= -4[6R-Z]$
$= -24R+4Z$

48. $-3\{2.1e-1.3[-f-2(e-5f)]\} = -3\{2.1e-1.3[-f-2e+10f]\}$
$= -3\{2.1e-1.3[-2e+9f]\}$
$= -3\{2.1e+2.6e-11.7f\}$
$= -3\{4.7e-11.7f\}$
$= -14.1e+35.1f$

49. $3D-(D-d) = 3D-D+d = 2D+d$

50. $i_1-(2-3i_2)+i_2 = i_1-2+3i_2+i_2 = i_1+4i_2-2$

51. $B+\frac{4}{3}\alpha \quad +2 \quad B-\frac{2}{3}\alpha \quad - \quad B+\frac{4}{3}\alpha \quad - \quad B-\frac{2}{3}\alpha \quad = \quad B+\frac{4}{3}\alpha+2B-\frac{4}{3}\alpha \quad - \quad B+\frac{4}{3}\alpha-B+\frac{2}{3}\alpha$
$= [3B] - \frac{6}{3}\alpha$
$= 3B-2\alpha$

52. Distance $= 30$ km/h $\times (t-1)$h $+ 40$ km/h $\times (t+2)$ h
$= 30(t-1)$ km $+ 40(t+2)$ km
$= (30t-30+40t+80)$ km
$= (70t+50)$ km

53. Memory $= x$(4 terabytes) + $(x+25)$(8 terabytes)
$= (4x+8x+200)$ terabytes
$= (12x+200)$ terabytes

54. Difference $= 2[(2n+1)(\$30)-(n-2)(\$20)]$

$$\begin{aligned} &= \$\ 2[60n+30-20n+40] \\ &= \$\ 2[40n+70] \\ &= \$\ (80n+140) \end{aligned}$$

55. **(a)** $(2x^2 - y + 2a) + (3y - x^2 - b) = 2x^2 - y + 2a + 3y - x^2 - b$

$$= x^2 + 2y + 2a - b$$

(b) $(2x^2 - y + 2a) - (3y - x^2 - b) = 2x^2 - y + 2a - 3y + x^2 + b$

$$= 3x^2 - 4y + 2a + b$$

56. $(3a^2 + b - c^3) + (2c^3 - 2b - a^2) - (4c^3 - 4b + 3) = 3a^2 + b - c^3 + 2c^3 - 2b - a^2 - 4c^3 + 4b - 3$

$$= 2a^2 + 3b - 3c^3 - 3$$

57. The final y should be added and the final 3 should be subtracted. The correct final answer is $-2x - 2y + 2$.

58. The final occurrence of $2c$ should be added rather than subtracted, resulting in the final answer of $7a - 6b - 2c$.

59. $|a-b| = |-(-a+b)|$

$$\begin{aligned} &= |-(b-a)| \\ &= |-1 \times (b-a)| \\ &= |-1| \times |(b-a)| \\ &= 1 \times |b-a| \\ &= |b-a| \end{aligned}$$

60. $(a-b)-c = a-b-c$

However, $a-(b-c) = a-b+c$

Since they are not equivalent, subtraction is not associative.

For example, $(10-5)-2 = 5-2 = 3$ is not the same as $10-(5-2) = 10-3 = 7$.

1.8 Multiplication of Algebraic Expressions

1. $2s^3(-st^4)^3(4s^2t) = 2s^3(-1)^3 s^3 t^{12}(4s^2t)$

$$\begin{aligned} &= -2s^6t^{12}(4s^2t) \\ &= -8s^8t^{13} \end{aligned}$$

2. $-2ax(3ax^2 - 4yz) = (-2ax)(3ax^2) - (-2ax)(4yz)$

$$\begin{aligned} &= (-6a^2x^3) - (-8axyz) \\ &= -6a^2x^3 + 8axyz \end{aligned}$$

3. $(x-2)(x-3) = x(x) + x(-3) + (-2)(x) + (-2)(-3)$

$$\begin{aligned} &= x^2 - 3x - 2x + 6 \\ &= x^2 - 5x + 6 \end{aligned}$$

4. $$\begin{aligned}(2a-b)^2 &= (2a-b)(2a-b)\\ &= (2a)(2a)+(2a)(-b)+(2a)(-b)+(-b)(-b)\\ &= 4a^2-2ab-2ab+b^2\\ &= 4a^2-4ab+b^2\end{aligned}$$

5. $(a^2)(ax) = a^3x$

6. $(2xy)(x^2y^3) = 2x^3y^4$

7. $-a^2c^2(a^2cx^3) = -a^4c^3x^3$

8. $$\begin{aligned}(-2cs^2)(-4cs)^2 &= (-2cs^2)(-4cs)(-4cs)\\ &= (-2cs^2)(16c^2s^2)\\ &= -32c^3s^4\end{aligned}$$

9. $$\begin{aligned}(2ax^2)^2(-2ax) &= (2ax^2)(2ax^2)(-2ax)\\ &= (4a^2x^4)(-2ax)\\ &= -8a^3x^5\end{aligned}$$

10. $$\begin{aligned}(6pq^3)(3pq^2)^2 &= (6pq^3)(3pq^2)(3pq^2)\\ &= (6pq^3)(9p^2q^4)\\ &= 54p^3q^7\end{aligned}$$

11. $$\begin{aligned}i^2(Ri+2ri) &= (i^2)(Ri)+(i^2)(2ri)\\ &= i^3R+2i^3r\end{aligned}$$

12. $$\begin{aligned}2x(-p-q) &= (2x)(-p)-(2x)(q)\\ &= -2px-2qx\end{aligned}$$

13. $$\begin{aligned}-3s(s^2-5t) &= (-3s)(s^2)+(-3s)(-5t)\\ &= -3s^3+15st\end{aligned}$$

14. $$\begin{aligned}-3b(2b^2-b) &= (-3b)(2b^2)+(-3b)(-b)\\ &= -6b^3+3b^2\end{aligned}$$

15. $$\begin{aligned}5m(m^2n+3mn) &= (5m)(m^2n)+(5m)(3mn)\\ &= 5m^3n+15m^2n\end{aligned}$$

16. $$\begin{aligned}a^2bc(2ac-3b^2c) &= (a^2bc)(2ac)+(a^2bc)(-3b^2c)\\ &= 2a^3bc^2-3a^2b^3c^2\end{aligned}$$

17. $$\begin{aligned}3M(-M-N+2) &= (3M)(-M)+(3M)(-N)+(3M)(2)\\ &= -3M^2-3MN+6M\end{aligned}$$

18. $$\begin{aligned}-4c^2(-9gc-2c+g^2) &= (-4c^2)(-9cg)+(-4c^2)(-2c)+(-4c^2)(g^2)\\ &= 36c^3g+8c^3-4c^2g^2\end{aligned}$$

19. $xy(tx^2)(x+y^3) = tx^3y(x+y^3)$
$= (tx^3y)(x) + (tx^3y)(y^3)$
$= tx^4y + tx^3y^4$

20. $-2(-3st^3)(3s-4t) = 6st^3(3s-4t)$
$= (6st^3)(3s) + (6st^3)(-4t)$
$= 18s^2t^3 - 24st^4$

21. $(x-3)(x+5) = (x)(x) + (x)(5) + (-3)(x) + (-3)(5)$
$= x^2 + 5x - 3x - 15$
$= x^2 + 2x - 15$

22. $(a+7)(a+1) = (a)(a) + (a)(1) + (7)(a) + (7)(1)$
$= a^2 + a + 7a + 7$
$= a^2 + 8a + 7$

23. $(x+5)(2x-1) = (x)(2x) + (x)(-1) + (5)(2x) + (5)(-1)$
$= 2x^2 - x + 10x - 5$
$= 2x^2 + 9x - 5$

24. $(4t_1+t_2)(2t_1-3t_2) = (4t_1)(2t_1) + (4t_1)(-3t_2) + (t_2)(2t_1) + (t_2)(-3t_2)$
$= 8t_1^2 - 12t_1t_2 + 2t_1t_2 - 3t_2^2$
$= 8t_1^2 - 10t_1t_2 - 3t_2^2$

25. $(y+8)(y-8) = (y)(y) + (y)(-8) + (8)(y) + (8)(-8)$
$= y^2 - 8y + 8y - 64$
$= y^2 - 64$

26. $(z-4)(z+4) = (z)(z) + (z)(4) + (-4)(z) + (-4)(4)$
$= z^2 + 4z - 4z - 16$
$= z^2 - 16$

27. $(2a-b)(-2b+3a) = (2a)(-2b) + (2a)(3a) + (-b)(-2b) + (-b)(3a)$
$= -4ab + 6a^2 + 2b^2 - 3ab$
$= 6a^2 - 7ab + 2b^2$

28. $(-3+4w^2)(3w^2-1) = (-3)(3w^2) + (-3)(-1) + (4w^2)(3w^2) + (4w)(-1)$
$= -9w^2 + 3 + 12w^4 - 4w^2$
$= 12w^4 - 13w^2 + 3$

29. $(2s+7t)(3s-5t) = (2s)(3s) + (2s)(-5t) + (7t)(3s) + (7t)(-5t)$
$= 6s^2 - 10st + 21st - 35t^2$
$= 6s^2 + 11st - 35t^2$

30. $(5p-2q)(p+8q)=(5p)(p)+(5p)(8q)+(-2q)(p)+(-2q)(8q)$
$=5p^2+40pq-2pq-16q^2$
$=5p^2+38pq-16q^2$

31. $(x^2-1)(2x+5)=(x^2)(2x)+(x^2)(5)+(-1)(2x)+(-1)(5)$
$=2x^3+5x^2-2x-5$

32. $(3y^2+2)(2y-9)=(3y^2)(2y)+(3y^2)(-9)+(2)(2y)+(-9)(2)$
$=6y^3-27y^2+4y-18$

33. $(x-2y-4)(x-2y+4)$
$=(x)(x)+(x)(-2y)+(x)(4)+(-2y)(x)+(-2y)(-2y)+(-2y)(4)+(-4)(x)+(-4)(-2y)+(-4)(4)$
$=x^2-2xy+4x-2xy+4y^2-8y-4x+8y-16$
$=x^2+4y^2-4xy-16$

34. $(2a+3b+1)(2a+3b-1)$
$=(2a)(2a)+(2a)(3b)+(2a)(-1)+(3b)(2a)+(3b)(3b)+(3b)(-1)+(1)(2a)+(1)(3b)+(1)(-1)$
$=4a^2+6ab-2a+6ab+9b^2-3b+2a+3b-1$
$=4a^2+9b^2+12ab-1$

35. $2(a+1)(a-9)=2[(a)(a)+(a)(-9)+(1)(a)+(-9)(1)]$
$=2[a^2-9a+a-9]$
$=2[a^2-8a-9]$
$=2a^2-16a-18$

36. $-5(y-3)(y+6)=-5[(y)(y)+(y)(6)+(-3)(y)+(-3)(6)]$
$=-5[y^2+6y-3y-18]$
$=-5[y^2+3y-18]$
$=-5y^2-15y+90$

37. $-3(3-2T)(3T+2)=-3[(3)(3T)+(3)(2)+(-2T)(3T)+(-2T)(2)]$
$=-3[-6T^2+9T-4T+6]$
$=-3[-6T^2+5T+6]$
$=18T^2-15T-18$

38. $2n(-n+5)(6n+5)=2n[(-n)(6n)+(-n)(5)+(5)(6n)+(5)(5)]$
$=2n[-6n^2-5n+30n+25]$
$=2n[-6n^2+25n+25]$
$=-12n^3+50n^2+50n$

39. $2L(L+1)(4-L)=2L[(L)(4)+(L)(-L)+(1)(4)+(1)(-L)]$
$=2L[-L^2+4L-L+4]$
$=2L[-L^2+3L+4]$
$=-2L^3+6L^2+8L$

40. $ax(x+4)(7-x^2) = ax[(x)(7)+(x)(-x^2)+(4)(7)+(4)(-x^2)]$
$= ax[-x^3-4x^2+7x+28]$
$= -ax^4-4ax^3+7ax^2+28ax$

41. $(3x-7)^2 = (3x-7)(3x-7)$
$= (3x)(3x)+(3x)(-7)+(-7)(3x)+(-7)(-7)$
$= 9x^2-21x-21x+49$
$= 9x^2-42x+49$

42. $(x-3y)^2 = (x-3y)(x-3y)$
$= (x)(x)+(x)(-3y)+(-3y)(x)+(-3y)(-3y)$
$= x^2-3xy-3xy+9y^2$
$= x^2-6xy+9y^2$

43. $(x_1+3x_2)^2 = (x_1+3x_2)(x_1+3x_2) = (x_1)(x_1)+(x_1)(3x_2)+(3x_2)(x_1)+(3x_2)(3x_2)$
$= x_1{}^2+3x_1x_2+3x_1x_2+9x_2{}^2$
$= x_1{}^2+6x_1x_2+9x_2{}^2$

44. $(-7m-1)^2 = (-7m-1)(-7m-1)$
$= (-7m)(-7m)+(-7m)(-1)+(-1)(-7m)+(-1)(-1)$
$= 49m^2+7m+7m+1$
$= 49m^2+14m+1$

45. $(xyz-2)^2 = (xyz-2)(xyz-2)$
$= (xyz)(xyz)+(xyz)(-2)+(-2)(xyz)+(-2)(-2)$
$= x^2y^2z^2-2xyz-2xyz+4$
$= x^2y^2z^2-4xyz+4$

46. $(-6x^2+b)^2 = (-6x^2+b)(-6x^2+b)$
$= (-6x^2)(-6x^2)+(-6x^2)(b)+(b)(-6x^2)+(b)(b)$
$= 36x^4-6bx^2-6bx^2+b^2$
$= 36x^4-12bx^2+b^2$

47. $2(x+8)^2 = 2[(x+8)(x+8)]$
$= 2[(x)(x)+(x)(8)+(8)(x)+(8)(8)]$
$= 2[x^2+8x+8x+64]$
$= 2[x^2+16x+64]$
$= 2x^2+32x+128$

48. $3(3R-4)^2 = 3[(3R-4)(3R-4)]$
$= 3[(3R)(3R)+(3R)(-4)+(-4)(3R)+(-4)(-4)]$
$= 3[9R^2-12R-12R+16]$
$= 3[9R^2-24R+16]$
$= 27R^2-72R+48$

49. $(2+x)(3-x)(x-1) = [(6-2x+3x-x^2)](x-1)$
$= (x-1)[-x^2+x+6]$
$= (x)(-x^2)+(x)(x)+(6)(x)+(-1)(-x^2)+(-1)(x)+(-1)(6)$
$= -x^3+x^2+6x+x^2-x-6$
$= -x^3+2x^2+5x-6$

50. $(-c^2+3x)^3 = (-c^2+3x)(-c^2+3x)(-c^2+3x)$
$= [(3x)(3x)-3c^2x-3c^2x+c^4)](-c^2+3x)$
$= (-c^2+3x)[9x^2-6c^2x+c^4]$
$= (-c^2)(9x^2)+(-c^2)(-6c^2x)+(-c^2)(+c^4)+(3x)(9x^2)+(3x)(-6c^2x)+(3x)(+c^4)$
$= -9c^2x^2+6c^4x-c^6+27x^3-18c^2x^2+3c^4x$
$= -c^6+9c^4x-27c^2x^2+27x^3$

51. $3T(T+2)(2T-1) = 3T[(T)(2T)+(T)(-1)+(2)(2T)+(2)(-1)]$
$= 3T[2T^2-T+4T-2]$
$= 3T[2T^2-T+4T-2]$
$= 3T[2T^2+3T-2]$
$= 6T^3+9T^2-6T$

52. $[(x-2)^2(x+2)]^2$
$= [(x-2)(x-2)(x+2)][(x-2)(x-2)(x+2)]$
$= [(x-2)[(x)(x)+(-2)(x)+(2)(x)+(-2)(2)]][(x-2)[(x)(x)+(-2)(x)+(2)(x)+(-2)(2)]]$
$= [(x-2)[x^2-2x+2x-4]][(x-2)[x^2-2x+2x-4]]$
$= [(x-2)[x^2-4]][(x-2)[x^2-4]]$
$= [(x)(x^2)+(-4)(x)+(-2)(x^2)+(-2)(-4)][(x)(x^2)+(-4)(x)+(-2)(x^2)+(-2)(-4)]$
$= [x^3-2x^2-4x+8][x^3-2x^2-4x+8]$
$= (x^3)(x^3)+(x^3)(-2x^2)+(x^3)(-4x)+(x^3)(8)+(-2x^2)(x^3)+(-2x^2)(-2x^2)+(-2x^2)(-4x)+(-2x^2)(8)$
$+(-4x)(x^3)+(-4x)(-2x^2)+(-4x)(-4x)+(-4x)(8)+(8)(x^3)+(8)(-2x^2)+(8)(-4x)+(8)(8)$
$= x^6-2x^5-4x^4+8x^3-2x^5+4x^4+8x^3-16x^2-4x^4+8x^3+16x^2-32x+8x^3-16x^2-32x+64$
$= x^6-4x^5-4x^4+32x^3-16x^2-64x+64$

53. **(a)** $(x+y)^2 = (3+4)^2 = 7^2 = 49$
$x^2+y^2 = 3^2+4^2 = 9+16 = 25$
$(x+y)^2 \neq x^2+y^2$
$49 \neq 25$

(b) $(x-y)^2=(3-4)^2=(-1)^2=1$

$x^2-y^2=3^2-4^2=9-16=-7$

$(x-y)^2 \neq x^2-y^2$

$1 \neq -7$

54. One can write $(x+3)^5=(x+3)(x+3)(x+3)(x+3)(x+3)$ and then perform the multiplications using the rightmost pair of terms at each step.

55.
$$\begin{aligned}(x+y)^3 &= (x+y)(x+y)(x+y)\\ &= (x+y)[(x)(x)+(x)(y)+(y)(x)+(y)(y)]\\ &= (x+y)[x^2+xy+xy+y^2]\\ &= (x+y)[x^2+2xy+y^2]\\ &= (x)(x^2)+(x)(2xy)+(x)(y^2)+(y)(x^2)+(y)(2xy)+(y)(y^2)\\ &= x^3+2x^2y+y^2x+x^2y+2y^2x+y^3\\ &= x^3+3x^2y+3y^2x+y^3\end{aligned}$$

This differs from x^3+y^3 by the presence of $3x^2y+3y^2x$ in the center.

56.
$$\begin{aligned}&(x+y)(x^2-xy+y^2)\\ &= (x)(x^2)+(x)(-xy)+(x)(y^2)+(y)(x^2)+(y)(-xy)+(y)(y^2)\\ &= x^3-x^2y+y^2x+x^2y-y^2x+y^3\\ &= x^3+y^3\end{aligned}$$

57.
$$\begin{aligned}P(1+0.01r)^2 &= P(1+0.01r)(1+0.01r)\\ &= P[(1)(1)+(1)(0.01r)+(0.01r)(1)+(0.01r)(0.01r)]\\ &= P[1+0.01r+0.01r+0.0001r^2]\\ &= 0.0001r^2P+0.02rP+P\end{aligned}$$

58.
$$\begin{aligned}1000(1+0.0025r)^2 &= 1000(1+0.0025r)(1+0.0025r)\\ &= 1000[(1)(1)+(1)(0.0025r)+(0.0025r)(1)+(0.0025r)(0.0025r)]\\ &= 1000[1+0.0025r+0.0025r+0.0000625r^2]\\ &= 1000+2.5r+0.0625r^2\end{aligned}$$

59. The room will be $5+w+5=w+10$ feet wide and $5+2w+5=2w+10$ feet long. Its area is

$$\begin{aligned}(w+10)(2w+10) &= (w)(2w)+(w)(10)+(10)(2w)+(10)(10)\\ &= 2w^2+10w+20w+100\\ &= 2w^2+30w+100\end{aligned}$$

60.
$$\begin{aligned}R &= xp\\ &= x(30-0.01x)\\ &= 30x-0.01x^2\end{aligned}$$

61. $(2R-X)^2-(R^2+X^2)=(2R-X)(2R-X)-(R^2+X^2)$
$$=[(2R)(2R)+(2R)(-X)+(2R)(-X)+(-X)(-X)]-(R^2+X^2)$$
$$=4R^2-2RX-2RX+X^2-R^2-X^2$$
$$=3R^2-4RX$$

62. $(2T^3+3)(T^2-T-3)=(2T^3)(T^2)+(2T^3)(-T)+(2T^3)(-3)+(3)(T^2)+(3)(-T)+(3)(-3)$
$$=2T^5-2T^4-6T^3+3T^2-3T-9$$

63. Number of switches for n elements $= n^2$

Number of switches for $n+100$ elements $=(n+100)^2$
$$=(n+100)(n+100)$$
$$=(n)(n)+(n)(100)+(100)(n)+(100)(100)$$
$$=n^2+100n+100n+10000$$
$$=n^2+200n+10000$$

64. $(T^2-100)(T-10)(T+10)=(T^2-100)\ \ T^2+10T-10T-100$
$$=(T^2-100)\ \ T^2-100$$
$$=T^4-100T^2-100T^2+10\ 000$$
$$=T^4-200T^2+10\ 000$$

65. $(R_1+R_2)^2-2R_2(R_1+R_2)=\left[(R_1+R_2)(R_1+R_2)\right]-2R_2(R_1+R_2)$
$$=\left[(R_1)(R_1)+(R_1)(R_2)+(R_2)(R_1)+(R_2)(R_2)\right]-2R_1R_2-2R_2^2$$
$$=\ \ R_1^2+R_1R_2+R_1R_2+R_2^2\ \ -2R_1R_2-2R_2^2$$
$$=R_1^2-R_2^2$$

66. $27x^2-24(x-6)^2-(x-12)^3$
$$=27x^2-24(x-6)(x-6)-(x-12)(x-12)(x-12)$$
$$=27x^2-24\ \ x^2-6x-6x+36\ \ -(x-12)\ \ x^2-12x-12x+144$$
$$=27x^2-24\ \ x^2-12x+36\ \ -(x-12)\ \ x^2-24x+144$$
$$=27x^2-24x^2+288x-864-\ \ (x)(x^2)+(x)(-24x)+(x)(144)+(-12)(x^2)+(-12)(-24x)+(-12)(144)$$
$$=3x^2+288x-864-x^3+24x^2-144x+12x^2-288x+1728$$
$$=-x^3+39x^2-144x+864$$

1.9 Division of Algebraic Expressions

1. $\dfrac{-6a^2xy^2}{-2a^2xy^5}=\left(\dfrac{-6}{-2}\right)\dfrac{a^{2-2}x^{1-1}}{y^{5-2}}=\dfrac{3}{y^3}$

2.
$$\frac{4x^3y-8x^3y^2+2x^2y}{2xy^2}=\frac{4x^3y}{2xy^2}-\frac{8x^3y^2}{2xy^2}+\frac{2x^2y}{2xy^2}$$
$$=\frac{2x^{3-1}}{y^{2-1}}-4x^{3-1}y^{2-2}+\frac{x^{2-1}}{y^{2-1}}$$
$$=\frac{2x^2}{y}-4x^2+\frac{x}{y}$$

3.
$$\begin{array}{r} 3x-2 \\ 2x-1\,\overline{)\,6x^2-7x+2} \\ \underline{6x^2-3x} \\ -4x+2 \\ \underline{-4x+2} \\ 0 \end{array}$$

4.
$$\begin{array}{r} 2x-1 \\ 4x^2-1\,\overline{)\,8x^3-4x^2+0x+3} \\ \underline{8x^3\qquad\quad -2x} \\ -4x^2+2x+3 \\ -\underline{4x^2\qquad\quad 1} \\ 2x+2 \end{array}$$
$$\frac{8x^3-4x^2+3}{4x^2-1}=2x-1+\frac{2x+2}{4x^2-1}$$

5. $$\frac{8x^3y^2}{-2xy}=-4x^{3-1}y^{2-1}=-4x^2y$$

6. $$\frac{-18b^7c^3}{bc^2}=-18b^{7-1}c^{3-2}=-18b^6c$$

7. $$\frac{-16r^3t^5}{-4r^5t}=\frac{4t^{5-1}}{r^{5-3}}=\frac{4t^4}{r^2}$$

8. $$\frac{51mn^5}{17m^2n^2}=\frac{3n^{5-2}}{m^{2-1}}=\frac{3n^3}{m}$$

9. $$\frac{(15x^2y)(2xz)}{10xy}=\frac{30x^3yz}{10xy}=3x^{3-1}y^{1-1}z=3x^2z$$

10. $$\frac{(5sT)(8s^2T^3)}{10s^3T^2}=\frac{40s^3T^4}{10s^3T^2}=4s^{3-3}T^{4-2}=4T^2$$

11. $$\frac{(4a^3)(2x)^2}{(4ax)^2}=\frac{4a^3(4x^2)}{16a^2x^2}=1a^{3-2}x^{2-2}=a$$

12. $\dfrac{12a^2b}{(3ab^2)^2}=\dfrac{12a^2b}{9a^2b^4}=\dfrac{4a^{2-2}}{3b^{4-1}}=\dfrac{4}{3b^3}$

13. $\dfrac{3a^2x+6xy}{3x}=\dfrac{3a^2x}{3x}+\dfrac{6xy}{3x}=\dfrac{3a^2x^{1-1}}{3}+\dfrac{6x^{1-1}y}{3}=a^2+2y$

14. $\dfrac{2m^2n-6mn}{-2m}=\dfrac{2m^2n}{-2m}-\dfrac{6mn}{-2m}=-m^{2-1}n+3m^{1-1}n=-mn+3n$

15. $\dfrac{3rst-6r^2st^2}{3rs}=\dfrac{3rst}{3rs}-\dfrac{6r^2st^2}{3rs}=r^{1-1}s^{1-1}t-2r^{2-1}s^{1-1}t^2=-2rt^2+t$

16. $\dfrac{-5a^2n-10an^2}{5an}=\dfrac{-5a^2n}{5an}-\dfrac{10an^2}{5an}=-a^{2-1}n^{1-1}-2a^{1-1}n^{2-1}=-a-2n$

17.
$$\begin{aligned}\frac{4pq^3+8p^2q^2-16pq^5}{4pq^2}&=\frac{4pq^3}{4pq^2}+\frac{8p^2q^2}{4pq^2}-\frac{16pq^5}{4pq^2}\\&=p^{1-1}q^{3-2}+2p^{2-1}q^{2-2}-4p^{1-1}q^{5-2}\\&=-4q^3+2p+q\end{aligned}$$

18.
$$\begin{aligned}\frac{a^2x_1x_2^2+ax_1^3-ax_1}{ax_1}&=\frac{a^2x_1x_2^2}{ax_1}+\frac{ax_1^3}{ax_1}-\frac{ax_1}{ax_1}\\&=a^{2-1}x_1^{1-1}x_2^2+a^{1-1}x_1^{3-1}-a^{1-1}x_1^{1-1}\\&=ax_2^2+x_1^2-1\end{aligned}$$

19.
$$\begin{aligned}\frac{2\pi fL-\pi fR^2}{\pi fR}&=\frac{2\pi fL}{\pi fR}-\frac{\pi fR^2}{\pi fR}\\&=\frac{2f^{1-1}L}{R}-f^{1-1}R^{2-1}\\&=\frac{2L}{R}-R\end{aligned}$$

20.
$$\begin{aligned}\frac{9(aB)^4-6aB^4}{-3aB^3}&=\frac{9(aB)^4}{-3aB^3}-\frac{6aB^4}{-3aB^3}\\&=-\frac{9a^4B^4}{3aB^3}+\frac{6aB^4}{3aB^3}\\&=-3a^{4-1}B^{4-3}+2a^{1-1}B^{4-3}\\&=-3a^3B+2B\end{aligned}$$

21.
$$\begin{aligned}\frac{-7a^2b+14ab^2-21a^3}{14a^2b^2}&=-\frac{7a^2b}{14a^2b^2}+\frac{14ab^2}{14a^2b^2}-\frac{21a^3}{14a^2b^2}\\&=-\frac{a^{2-2}}{2b^{2-1}}+\frac{b^{2-2}}{a^{2-1}}-\frac{3}{2}a^{3-2}b^{-2}\\&=-\frac{1}{2b}+\frac{1}{a}-\frac{3a}{2b^2}\end{aligned}$$

22. $\dfrac{2x^{n+2}+4ax^n}{2x^n}=\dfrac{2x^{n+2}}{2x^n}+\dfrac{4ax^n}{2x^n}$

$=x^{n-n+2}+2ax^{n-n}$

$=x^2+2a$

23. $\dfrac{6y^{2n}-4ay^{n+1}}{2y^n}=\dfrac{6y^{2n}}{2y^n}-\dfrac{4ay^{n+1}}{2y^n}$

$=3y^{2n-n}-2ay^{n-n+1}$

$=3y^n-2ay$

24. $\dfrac{3a(F+T)b^2-(F+T)}{a(F+T)}=\dfrac{3a(F+T)b^2}{a(F+T)}-\dfrac{(F+T)}{a(F+T)}$

$=\dfrac{3a^{1-1}\cancel{(F+T)}b^2}{\cancel{(F+T)}}-\dfrac{\cancel{(F+T)}}{a\cancel{(F+T)}}$

$=3b^2-\dfrac{1}{a}$

25. $x+4\overline{)x^2+9x+20}$, quotient $x+5$

x^2+4x

$5x+20$

$5x+20$

0

$\dfrac{x^2+9x+20}{x+4}=x+5$

26. $x-2\overline{)x^2+7x-18}$, quotient $x+9$

x^2-2x

$9x-18$

$9x-18$

0

$\dfrac{x^2+7x-18}{x-2}=x+9$

27.
$$\begin{array}{r} 2x+1 \\ x+3\overline{)2x^2+7x+3} \\ \underline{2x^2+6x} \\ x+3 \\ \underline{x+3} \\ 0 \end{array}$$

$$\frac{2x^2+7x+3}{x+3}=2x+1$$

28.
$$\begin{array}{r} 3t-4 \\ t-1\overline{)3t^2-7t+4} \\ \underline{3t^2-3t} \\ -4t+4 \\ \underline{-4t+4} \\ 0 \end{array}$$

$$\frac{3t^2-7t+4}{t-1}=3t-4$$

29.
$$\begin{array}{r} x-1 \\ x-2\overline{)x^2-3x+2} \\ \underline{x^2-2x} \\ -x+2 \\ \underline{-x+2} \\ 0 \end{array}$$

$$\frac{x^2-3x+2}{x-2}=x-1$$

30.
$$\begin{array}{r} 2x-7 \\ x+1\overline{)2x^2-5x-7} \\ \underline{2x^2+2x} \\ -7x-7 \\ \underline{-7x-7} \\ 0 \end{array}$$

$$\frac{2x^2-5x-7}{x+1}=2x-7$$

31.
$$\begin{array}{r} 4x^2-x-1 \\ 2x-3\overline{)8x^3-14x^2+x+0} \\ \underline{8x^3-12x^2} \\ -2x^2+x \\ \underline{-2x^2+3x} \\ -2x+0 \\ \underline{-2x+3} \\ -3 \end{array}$$

$$\frac{8x^3-14x^2+x}{2x-3}=4x^2-x-1-\frac{3}{2x-3}$$

32.
$$\begin{array}{r} 3y+2 \\ 2y+1\overline{)6y^2+7y+6} \\ \underline{6y^2+3y} \\ 4y+6 \\ \underline{4y+2} \\ 4 \end{array}$$

$$\frac{6y^2+7y+6}{2y+1}=3y+2+\frac{4}{2y+1}$$

33.
$$\begin{array}{r} Z-2 \\ 4Z+3\overline{)4Z^2-5Z-7} \\ \underline{4Z^2+3Z} \\ -8Z-7 \\ \underline{-8Z-6} \\ -1 \end{array}$$

$$\frac{4Z^2-5Z-7}{4Z+3}=Z-2-\frac{1}{4Z+3}$$

34.
$$\begin{array}{r} 2x+1 \\ 3x-4\overline{)6x^2-5x-9} \\ \underline{6x^2-8x} \\ 3x-9 \\ \underline{3x-4} \\ -5 \end{array}$$

$$\frac{6x^2-5x-9}{3x-4}=2x+1-\frac{5}{3x-4}$$

35.

$$\begin{array}{r} x^2+x-6 \\ x+2\overline{\smash{)}x^3+3x^2-4x-12} \\ \underline{x^3+2x^2} \\ x^2-4x \\ \underline{x^2+2x} \\ -6x-12 \\ \underline{-6x-12} \\ 0 \end{array}$$

$$\frac{x^3+3x^2-4x-12}{x+2}=x^2+x-6$$

36.

$$\begin{array}{r} x^2+7x+9 \\ 3x-2\overline{\smash{)}3x^3+19x^2+13x-20} \\ \underline{3x^3-2x^2} \\ 21x^2+13x \\ \underline{21x^2-14x} \\ 27x-20 \\ \underline{27x-18} \\ -2 \end{array}$$

$$\frac{3x^3+19x^2+13x-20}{3x-2}=x^2+7x+9-\frac{2}{3x-2}$$

37.

$$\begin{array}{r} 2a^2+8 \\ a^2-2\overline{\smash{)}2a^4+0a^3+4a^2+0a-16} \\ \underline{2a^4-4a^2} \\ 8a^2-16 \\ \underline{8a^2-16} \\ 0 \end{array}$$

$$\frac{2a^4+4a^2-16}{a^2-2}=2a^2+8$$

38.

$$\begin{array}{r} 2T+1 \\ 3T^2-T+2\overline{\smash{)}6T^3+T^2+0T+2} \\ \underline{6T^3-2T^2+4T} \\ 3T^2-4T+2 \\ \underline{3T^2-T+2} \\ -3T \end{array}$$

$$\frac{6T^3+T^2+2}{3T^2-T+2}=2T+1-\frac{3T}{3T^2-T+2}$$

39.
$$\begin{array}{rl} & y^2-3y+9 \\ y+3 & \overline{)\,y^3+0y^2+0y+27} \\ & \underline{y^3+3y^2} \\ & -3y^2+0y \\ & \underline{-3y^2-9y} \\ & 9y+27 \\ & \underline{9y+27} \\ & 0 \end{array}$$

$$\frac{y^3+27}{y+3}=y^2-3y+9$$

40.
$$\begin{array}{rl} & D^2+D+1 \\ D-1 & \overline{)\,D^3+0D^2+0D-1} \\ & \underline{D^3-1D^2} \\ & D^2+0D \\ & \underline{D^2-1D} \\ & D-1 \\ & \underline{D-1} \\ & 0 \end{array}$$

$$\frac{D^3-1}{D-1}=D^2+D+1$$

41.
$$\begin{array}{rl} & x-y \\ x-y & \overline{)\,x^2-2xy+y^2} \\ & \underline{x^2-xy} \\ & -xy+y^2 \\ & \underline{-xy+y^2} \\ & 0 \end{array}$$

$$\frac{x^2-2xy+y^2}{x-y}=x-y$$

42.
$$\begin{array}{rl} & 3r+4R \\ r-3R & \overline{)\,3r^2-5rR+2R^2} \\ & \underline{3r^2-9rR} \\ & 4rR+2R^2 \\ & \underline{4rR-12R^2} \\ & 14R^2 \end{array}$$

$$\frac{3r^2-5rR+2R^2}{r-3R}=3r+4R+\frac{14R^2}{r-3R}$$

43.
$$\begin{array}{r} t-2 \\ t^2+2t+4\overline{)t^3+0t^2+0t-8} \\ \underline{t^3+2t^2+4t} \\ -2t^2-4t-8 \\ \underline{-2t^2-4t-8} \\ 0 \end{array}$$

$$\frac{t^3-8}{t^2+2t+4}=t-2$$

44.
$$\begin{array}{r} a^2+2ab+2b^2 \\ a^2-2ab+2b^2\overline{)a^4+0a^3b+0a^2b^2+0ab^3+b^4} \\ \underline{a^4-2a^3b+2a^2b^2} \\ 2a^3b-2a^2b^2+0ab^3 \\ \underline{2a^3b-4a^2b^2+4ab^3} \\ 2a^2b^2-4ab^3+b^4 \\ \underline{2a^2b^2-4ab^3+4b^4} \\ -3b^4 \end{array}$$

$$\frac{a^4+b^4}{a^2-2ab+2b^2}=a^2+2ab+2b^2-\frac{3b^4}{a^2-2ab+2b^2}$$

45. We know that $2x+1$ multiplied by $x+c$ will give us $2x^2-9x-5$, so $2x^2-9x-5$ divided by $2x+1$ will give us $x+c$:

$$\begin{array}{r} x-5 \\ 2x+1\overline{)2x^2-9x-5} \\ \underline{2x^2+x} \\ -10x-5 \\ \underline{-10x-5} \\ 0 \end{array}$$

$$x+c=x-5$$
$$c=-5$$

46.
$$\begin{array}{r} 2x-3 \\ 3x+4\overline{)6x^2-x+k} \\ \underline{6x^2+8x} \\ -9x+k \\ \underline{-9x-12} \\ 0 \end{array}$$

$$k-(-12)=0$$
$$k+12=0$$
$$k=-12$$

47.
$$
\begin{array}{r}
x^3 - x^2 + x - 1 \\
x+1 \overline{\big)\, x^4 + 0x^3 + 0x^2 + 0x + 1} \\
\underline{x^4 + x^3} \qquad\qquad\qquad \\
-x^3 + 0x^2 \qquad\qquad \\
\underline{-x^3 - x^2} \qquad\qquad \\
x^2 + 0x \qquad \\
\underline{x^2 + x} \qquad \\
-x + 1 \\
\underline{-x - 1} \\
2
\end{array}
$$

$$\frac{x^4+1}{x+1} = x^3 - x^2 + x - 1 + \frac{2}{x+1} \neq x^3$$

48.
$$
\begin{array}{r}
x^2 - xy + y^2 \\
x+y \overline{\big)\, x^3 + 0x^2y + 0y^2x + 0x + y^3} \\
\underline{x^3 + x^2\text{y}} \qquad\qquad\qquad\qquad \\
-x^2\text{y} + 0y^2x \qquad\qquad \\
\underline{-x^2y - y^2x} \qquad\qquad \\
y^2x + y^3 \\
\underline{y^2x + y^3} \\
0
\end{array}
$$

$$\frac{x^3+y^3}{x+y} = x^2 - xy + y \neq x^2 + y^2$$

49.
$$
\begin{aligned}
V_1 \;\; 1 + \frac{T_2 - T_1}{T_1} \;\; &= V_1 \;\; 1 + \frac{T_2}{T_1} - \frac{T_1}{T_1} \\
&= V_1 \;\; 1 + \frac{T_2}{T_1} - 1 \\
&= V_1 \frac{T_2}{T_1} \\
&= \frac{V_1T_2}{T_1}
\end{aligned}
$$

50.
$$
\begin{array}{r}
3x + 2 \\
2x+5 \overline{\big)\, 6x^2 + 19x + 10} \\
\underline{6x^2 + 15x} \qquad \\
4x + 10 \\
\underline{4x + 10} \\
0
\end{array}
$$

The width is $3x + 2$

51. $$\frac{8A^5 + 4A^3\mu^2E^2 - A\mu^4E^4}{8A^4} = \frac{8A^5}{8A^4} + \frac{4A^3\mu^2E^2}{8A^4} - \frac{A\mu^4E^4}{8A^4}$$
$$= A^{5-4} + \frac{\mu^2E^2}{2A^{4-3}} - \frac{\mu^4E^4}{8A^{4-1}}$$
$$= A + \frac{\mu^2E^2}{2A} - \frac{\mu^4E^4}{8A^3}$$

52. $$\frac{6R_1 + 6R_2 + R_1R_2}{6R_1R_2} = \frac{6R_1}{6R_1R_2} + \frac{6R_2}{6R_1R_2} + \frac{R_1R_2}{6R_1R_2}$$
$$= \frac{\cancel{R_1}}{\cancel{R_1}R_2} + \frac{\cancel{R_2}}{R_1\cancel{R_2}} + \frac{\cancel{R_1R_2}}{6\cancel{R_1R_2}}$$
$$= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{6}$$

53. $$\frac{GMm[(R+r)-(R-r)]}{2rR} = \frac{GMm[R+r-R+r]}{2rR}$$
$$= \frac{GMm[2r]}{2rR}$$
$$= \frac{GMm\,[\cancel{2r}]}{\cancel{2r}R}$$
$$= \frac{GMm}{R}$$

54.
$$\begin{array}{r l}
 & 3T^2 - 2T - 4 \\
T-2 \,\big) & 3T^3 - 8T^2 + 0T + 8 \\
 & \underline{3T^3 - 6T^2} \\
 & \quad -2T^2 + 0T \\
 & \quad \underline{-2T^2 + 4T} \\
 & \qquad -4T + 8 \\
 & \qquad \underline{-4T + 8} \\
 & \qquad\qquad 0
\end{array}$$

55. $$\frac{s^2-2s-2}{s^4+4}^{-1}=\frac{s^4+4}{s^2-2s-2}$$

$$\begin{array}{r} s^2+2s+6 \\ s^2-2s-2\overline{\smash{)}s^4+0s^3+0s^2+0s+4} \\ \underline{s^4-2s^3-2s^2} \\ 2s^3+2s^2+0s \\ \underline{2s^3-4s^2-4s} \\ 6s^2+4s+4 \\ \underline{6s^2-12s-12} \\ 16s+16 \end{array}$$

$$\frac{s^4+4}{s^2-2s-2}=s^2+2s+6+\frac{16s+16}{s^2-2s-2}$$

56. $$\begin{array}{r} t^2-3t+5 \\ 2t+100\overline{\smash{)}2t^3+94t^2-290t+500} \\ \underline{2t^3+100t^2} \\ -6t^2-290t \\ \underline{-6t^2-300t} \\ 10t+500 \\ \underline{10t+500} \\ 0 \end{array}$$

1.10 Solving Equations

1. **(a)**
$$\begin{aligned} x-3&=-12 \\ x-3+3&=-12+3 \\ x&=-9 \end{aligned}$$

(b)
$$\begin{aligned} x+3&=-12 \\ x+3-3&=-12-3 \\ x&=-15 \end{aligned}$$

(c)
$$\begin{aligned} \frac{x}{3}&=-12 \\ 3\ \frac{x}{3}&=3(-12) \\ x&=-36 \end{aligned}$$

(d) $3x = -12$

$$\frac{3x}{3} = \frac{-12}{3}$$

$$x = -4$$

2.
$$7 - 2t = 9$$
$$7 - 7 - 2t = 9 - 7$$
$$-2t = 2$$
$$\frac{-2t}{-2} = \frac{2}{-2}$$
$$t = -1$$

Check:
$$7 - 2t = 9$$
$$7 - 2(-1) = 9$$
$$7 - (-2) = 9$$
$$9 = 9$$

3.
$$x - 7 = 3x - (8 - 6x)$$
$$x - 7 = 3x - 8 + 6x$$
$$x - 7 = 9x - 8$$
$$-8x = -1$$
$$x = \frac{1}{8}$$

4.
$$\frac{L}{3.80} = \frac{7}{4}$$
$$3.80\left(\frac{L}{3.80}\right) = 3.80\left(\frac{7}{4}\right)$$
$$L = 6.65 \text{ m}$$

5.
$$x - 2 = 7$$
$$x = 7 + 2$$
$$x = 9$$

6.
$$x - 4 = -1$$
$$x = -1 + 4$$
$$x = 3$$

7.
$$x + 5 = 4$$
$$x = 4 - 5$$
$$x = -1$$

8.
$$s + 6 = -3$$
$$s = -3 - 6$$
$$s = -9$$

9. $\frac{t}{2} = -5$

$t = 2(-5)$

$t = -10$

10. $\frac{x}{-4} = 2$

$x = -4(2)$

$x = -8$

11. $\frac{y-8}{3} = 4$

$y - 8 = 4(3)$

$y = 12 + 8$

$y = 20$

12. $\frac{7-r}{6} = 3$

$7 - r = 6(3)$

$-r = 18 - 7$

$-r = 11$

$r = -11$

13. $4E = -20$

$E = \frac{-20}{4}$

$E = -5$

14. $2x = 12$

$x = \frac{12}{2}$

$x = 6$

15. $5t + 9 = -1$

$5t = -1 - 9$

$t = \frac{-10}{5}$

$t = -2$

16. $5D - 2 = 13$

$5D = 13 + 2$

$D = \frac{15}{5}$

$D = 3$

17. $$\begin{aligned} 5-2y &= -3 \\ -2y &= -3-5 \\ y &= \frac{-8}{-2} \\ y &= 4 \end{aligned}$$

18. $$\begin{aligned} -5t+8 &= 18 \\ -5t &= 18-8 \\ t &= \frac{10}{-5} \\ t &= -2 \end{aligned}$$

19. $$\begin{aligned} 3x+7 &= x \\ x-3x &= 7 \\ -2x &= 7 \\ x &= -\frac{7}{2} \end{aligned}$$

20. $$\begin{aligned} 6+4L &= 5-3L \\ 4L+3L &= 5-6 \\ 7L &= -1 \\ L &= -\frac{1}{7} \end{aligned}$$

21. $$\begin{aligned} 2(3q+4) &= 5q \\ 6q+8 &= 5q \\ 6q-5q &= -8 \\ q &= -8 \end{aligned}$$

22. $$\begin{aligned} 3(4-n) &= -n \\ -n &= 12-3n \\ -n+3n &= 12 \\ 2n &= 12 \\ n &= \frac{12}{2} \\ n &= 6 \end{aligned}$$

23. $$\begin{aligned} -(r-4) &= 6+2r \\ -r+4 &= 6+2r \\ -r-2r &= 2 \\ -3r &= 2 \\ r &= -\frac{2}{3} \end{aligned}$$

24. $-(x+2)+5=5x$

$$5x=5-x-2$$
$$5x+x=3$$
$$6x=3$$
$$x=\frac{3}{6}=\frac{1}{2}$$

25. $8(y-5)=-2y$

$$8y-40=-2y$$
$$8y+2y=40$$
$$10y=40$$
$$y=\frac{40}{10}$$
$$y=4$$

26. $4(7-F)=-7$

$$28-4F=-7$$
$$-4F=-7-28$$
$$F=\frac{-35}{-4}=\frac{35}{4}$$

27. $0.1x-0.5(x-2)=2$

$$x-5(x-2)=2(10)$$
$$x-5x+10=20$$
$$-4x=20-10$$
$$x=\frac{10}{-4}=-\frac{5}{2}$$

28. $1.5x-0.3(x-4)=6$

$$15x-3(x-4)=6(10)$$
$$15x-3x+12=60$$
$$12x=60-12$$
$$x=\frac{48}{12}$$
$$x=4$$

29. $-4-3(1-2p)=-7+2p$

$$-4-3+6p=-7+2p$$
$$-7+6p-2p=-7$$
$$4p=-7+7$$
$$p=\frac{0}{4}$$
$$p=0$$

30. $$\begin{aligned} 3-6(2-3t) &= t-5 \\ 3-12+18t &= t-5 \\ -9+18t-t &= -5 \\ 17t &= 4 \\ t &= \frac{4}{17} \end{aligned}$$

31. $$\begin{aligned} \frac{4x-2(x-4)}{3} &= 8 \\ 4x-2x+8 &= 3(8) \\ 2x &= 24-8 \\ x &= \frac{16}{2} \\ x &= 8 \end{aligned}$$

32. $$\begin{aligned} 2x &= \frac{-5(7-3x)+2}{4} \\ 4(2x) &= -35+15x+2 \\ 8x-15x &= -33 \\ -7x &= -33 \\ x &= \frac{-33}{-7} = \frac{33}{7} \end{aligned}$$

33. $$\begin{aligned} |x|-9 &= 2 \\ |x| &= 2+9 = 11 \\ x &= 11 \text{ or } x = -11 \end{aligned}$$

34. $$\begin{aligned} 2-|x| &= 4 \\ -|x| &= 4-2 \\ |x| &= \frac{2}{-1} \\ |x| &= -2 \end{aligned}$$

There is no real solution for x.

35. $|2x-3| = 5$

$2x-3=5$ or $2x-3=-5$

$$\begin{aligned} 2x-3 &= 5 & 2x-3 &= -5 \\ 2x &= 5+3 & 2x &= -5+3 \\ 2x &= 8 & 2x &= -2 \\ x &= \frac{8}{2} & x &= \frac{-2}{2} \end{aligned}$$

$x = 4$ or $x = -1$

36. $|7-x|=1$

$7-x=1$ or $7-x=-1$

$-x=1-7 \qquad -x=-1-7$

$-x=-6 \qquad -x=-8$

$x=6$ or $x=8$

37. $5.8-0.3(x-6.0)=0.5x$

$0.5x=5.8-0.3x+1.8$

$0.5x+0.3x=7.6$

$0.8x=7.6$

$x=\dfrac{7.6}{0.8}$

$x=9.5$

38. $1.9t=0.5(4.0-t)-0.8$

$1.9t=2.0-0.5t-0.8$

$1.9t+0.5t=1.2$

$2.4t=1.2$

$t=\dfrac{1.2}{2.4}$

$t=0.50$

39. $-0.24(C-0.50)=0.63$

$-0.24C+0.12=0.63$

$-0.24C=0.63-0.12$

$-0.24C=0.51$

$C=\dfrac{0.51}{-0.24}$

$C=-2.125$

$C=-2.1$

40. $27.5(5.17-1.44x)=73.4$

$142.175-39.6x=73.4$

$-39.6x=73.4-142.175$

$-39.6x=-68.775$

$x=\dfrac{-68.775}{-39.6}$

$x=1.736742424$

$x=1.74$

41. $\dfrac{x}{2.0}=\dfrac{17}{6.0}$

$x=2.0\ \dfrac{17}{6.0}$

$x=5.6666666...$

$x=5.7$

42. $\frac{3.0}{7.0} = \frac{R}{42}$

$R = 42\ \frac{3.0}{7.0}$

$R = 18$

43. $\frac{165}{223} = \frac{13V}{15}$

$\frac{15}{13}\ \frac{165}{223} = \frac{15}{13}\ \frac{13V}{15}$

$V = \frac{2475}{2899}$

$V = 0.85374267$

$V = 0.85$

44. $\frac{276x}{17.0} = \frac{1360}{46.4}$

$276x = 17\ \frac{1360}{46.4}$

$x = \frac{498.2758621}{276}$

$x = 1.805347326$

$x = 1.81$

45. **(a)** $2x + 3 = 3 + 2x$

$2x + 3 = 2x + 3$

Is an identity, since it is true for all values of x.

(b) $2x - 3 = 3 - 2x$

$4x = 6$

$x = \frac{6}{4} = \frac{3}{2}$

Is conditional as x has one answer only.

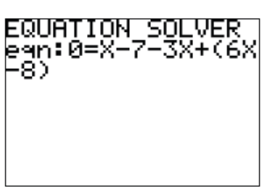
46. There are no values of a that result in a conditional equation. If $a = 0$, then the identity $2x = 2x$ results. If $a \neq 0$, then a contradiction results.

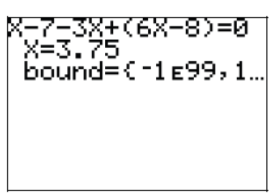
47. $x - 7 = 3x - (6x - 8)$

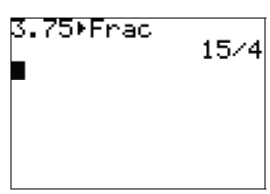
$0 = 3x - 6x + 8 - x + 7$

$0 = -4x + 15$

$x = 3.75$







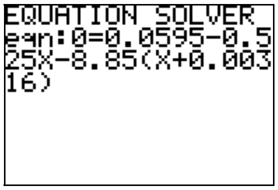
48. $0.0595 - 0.525i - 8.85(i + 0.0316) = 0$

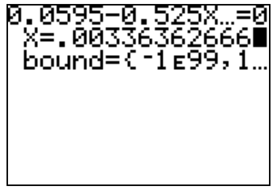
$0.595 - 0.525i - 8.85i - 0.27966 = 0$

$-9.375i + 0.31534 = 0$

$i = 0.033636266$

$i = 0.0336$





49. $0.03x + 0.06(2000 - x) = 96$

$0.03x + 120 - 0.06x = 96$

$-0.03x = 96 - 120$

$-0.03x = -24$

$x = \dfrac{-24}{-0.03}$

$x = \$800$

50. $15(5.5 + v) = 24(5.5 - v)$

$82.5 + 15v = 132 - 24v$

$15v + 24v = 132 - 82.5$

$39v = 49.5$

$v = \dfrac{49.5}{39}$

$v = 1.269230769$ km/h

$v = 1.3$ km/h

51. $1.1 = \dfrac{(T - 76)}{40}$

$40(1.1) = T - 76$

$44 = T - 76$

$T = 44 + 76$

$T = 120$ °C

52. $1.12V - 0.67(10.5 - V) = 0$

$1.12V - 7.035 + 0.67V = 0$

$1.79V - 7.035 = 0$

$1.79V = 7.035$

$V = \dfrac{7.035}{1.79}$

$V = 3.930167598$ V

$V = 3.9$ V

53.
$$\begin{aligned} 0.14n + 0.06(2000 - n) &= 0.09(2000) \\ 0.14n + 120 - 0.06n &= 180 \\ 0.14n - 0.06n &= 180 - 120 \\ 0.08n &= 60 \\ n &= \frac{60}{0.08} \\ n &= 750 \text{ L} \end{aligned}$$

54.
$$\begin{aligned} 210(3x) &= 55.3x + 38.5(8.25 - 3x) \\ 630x &= 55.3x + 317.625 - 115.5x \\ 630x - 55.3x + 115.5x &= 317.625 \\ 690.2x &= 317.625 \\ x &= \frac{317.625}{690.2} \\ x &= 0.4601927 \text{ m} \\ x &= 0.460 \text{ m} \end{aligned}$$

55.
$$\begin{aligned} \frac{x}{350 \text{ mi}} &= \frac{30 \text{ kW}\cdot\text{h}}{107 \text{ mi}} \\ x &= 350 \text{ mi} \times \frac{30 \text{ kW}\cdot\text{h}}{107 \text{ mi}} \\ x &= 98 \text{ kW}\cdot\text{h} \end{aligned}$$

56.
$$\begin{aligned} \frac{20 \text{ min}}{250 \text{ cal}} &= \frac{x}{400 \text{ cal}} \\ x &= 400 \text{ cal} \ \frac{20 \text{ min}}{250 \text{ cal}} \\ x &= 32 \text{ min} \end{aligned}$$

1.11 Formulas and Literal Equations

1.
$$\begin{aligned} v &= v_0 + at \\ v - v_0 &= at \\ a &= \frac{v - v_0}{t} \end{aligned}$$

2.
$$\begin{aligned} W &= \frac{L(wL + 2P)}{8} \\ 8W &= L(wL + 2P) \\ 8W &= wL^2 + 2LP \\ wL^2 &= 8W - 2LP \\ w &= \frac{8W - 2LP}{L^2} \end{aligned}$$

3.
$$V = V_0[1 + b(T - T_0)]$$
$$V = V_0[1 + bT - bT_0]$$
$$V = V_0 + bTV_0 - bT_0V_0$$
$$bT_0V_0 = V_0 + bTV_0 - V$$
$$T_0 = \frac{V_0 + bTV_0 - V}{bV_0}$$

4.
$$V = V_0 + V_0\beta T$$
$$V - V_0 = V_0\beta T$$
$$\beta = \frac{V - V_0}{V_0 T}$$

5.
$$E = IR$$
$$R = \frac{E}{I}$$

6.
$$pV = nRT$$
$$T = \frac{pV}{nR}$$

7.
$$rL = g_2 - g_1$$
$$g_1 + rL = g_2$$
$$g_1 = g_2 - rL$$

8.
$$W = S_d T - Q$$
$$Q + W = S_d T$$
$$Q = S_d T - W$$

9.
$$B = \frac{nTWL}{12}$$
$$12B = nTWL$$
$$n = \frac{12B}{TWL}$$

10.
$$P = 2\pi Tf$$
$$T = \frac{P}{2\pi f}$$

11.
$$p = p_a + dgh$$
$$p - p_a = dgh$$
$$h = \frac{p - p_a}{dg}$$

12.
$$2Q = 2I + A + S$$
$$2I = 2Q - A - S$$
$$I = \frac{2Q - A - S}{2}$$

13.
$$F_c = \frac{mv^2}{r}$$
$$rF_c = mv^2$$
$$r = \frac{mv^2}{F_c}$$

14.
$$P = \frac{4F}{\pi D^2}$$
$$P\pi D^2 = 4F$$
$$F = \frac{P\pi D^2}{4}$$

15.
$$S_T = \frac{A}{5T} + 0.05d$$
$$S_T - 0.05d = \frac{A}{5T}$$
$$A = 5T\left(S_T - 0.05d\right)$$

16.
$$u = -\frac{eL}{2m}$$
$$eL = -2mu$$
$$L = -\frac{2mu}{e}$$

17.
$$ct^2 = 0.3t - ac$$
$$ac + ct^2 = 0.3t$$
$$ac = 0.3t - ct^2$$
$$a = \frac{-ct^2 + 0.3t}{c}$$

18.
$$2p + dv^2 = 2d(C - W)$$
$$2p + dv^2 = 2Cd - 2dW$$
$$2Cd = dv^2 + 2p + 2dW$$
$$C = \frac{dv^2 + 2dW + 2p}{2d}$$

19.
$$T = \frac{c + d}{v}$$
$$c + d = Tv$$
$$d = Tv - c$$

20.
$$B = \frac{\mu_0 I}{2\pi R}$$
$$BR = \frac{\mu_0 I}{2\pi}$$
$$R = \frac{\mu_0 I}{2\pi B}$$

21.
$$\frac{K_1}{K_2} = \frac{m_1 + m_2}{m_1}$$
$$K_2(m_1 + m_2) = K_1 m_1$$
$$K_2 m_1 + K_2 m_2 = K_1 m_1$$
$$K_2 m_2 = K_1 m_1 - K_2 m_1$$
$$m_2 = \frac{K_1 m_1 - K_2 m_1}{K_2}$$

22.
$$f = \frac{F}{d - F}$$
$$f(d - F) = F$$
$$fd - fF = F$$
$$fd = F + fF$$
$$d = \frac{F + fF}{f}$$

23.
$$a = \frac{2mg}{M + 2m}$$
$$a(M + 2m) = 2gm$$
$$aM + 2am = 2gm$$
$$aM = 2gm - 2am$$
$$M = \frac{2gm - 2am}{a}$$

24.
$$v = \frac{V(m + M)}{m}$$
$$mv = mV + MV$$
$$MV = mv - mV$$
$$M = \frac{mv - mV}{V}$$

25.
$$C_0^2 = C_1^2(1 + 2V)$$
$$C_0^2 = C_1^2 + 2C_1^2 V$$
$$2C_1^2 V = C_0^2 - C_1^2$$
$$V = \frac{C_0^2 - C_1^2}{2C_1^2}$$

26.
$$\begin{aligned} A_1 &= A(M+1) \\ A_1 &= AM + A \\ AM &= A_1 - A \\ M &= \frac{A_1 - A}{A} \end{aligned}$$

27.
$$\begin{aligned} N &= r(A-s) \\ N &= Ar - rs \\ rs + N &= Ar \\ rs &= Ar - N \\ s &= \frac{Ar - N}{r} \end{aligned}$$

28.
$$\begin{aligned} T &= 3(T_2 - T_1) \\ T &= 3T_2 - 3T_1 \\ 3T_1 + T &= 3T_2 \\ 3T_1 &= 3T_2 - T \\ T_1 &= \frac{3T_2 - T}{3} \end{aligned}$$

29.
$$\begin{aligned} T_2 &= T_1 - \frac{h}{100} \\ 100T_2 &= 100T_1 - h \\ h + 100T_2 &= 100T_1 \\ h &= 100T_1 - 100T_2 \end{aligned}$$

30.
$$\begin{aligned} p_2 &= p_1 + rp_1(1-p_1) \\ p_2 - p_1 &= rp_1(1-p_1) \\ r &= \frac{p_2 - p_1}{p_1(1-p_1)} \end{aligned}$$

31.
$$\begin{aligned} Q_1 &= P(Q_2 - Q_1) \\ Q_1 &= PQ_2 - PQ_1 \\ PQ_2 &= Q_1 + PQ_1 \\ Q_2 &= \frac{Q_1 + PQ_1}{P} \end{aligned}$$

32.
$$\begin{aligned} p - p_a &= dg\left(y_2 - y_1\right) \\ y_2 - y_1 &= \frac{p - p_a}{dg} \\ -y_1 &= \frac{p - p_a}{dg} - y_2 \\ y_1 &= y_2 - \frac{p - p_a}{dg} \end{aligned}$$

33.
$$N = N_1T - N_2(1-T)$$
$$N_1T = N + N_2(1-T)$$
$$N_1 = \frac{N + N_2 - N_2T}{T}$$

34.
$$t_a = t_c + (1-h)t_m$$
$$t_a = t_c + t_m - ht_m$$
$$t_a + ht_m = t_c + t_m$$
$$ht_m = t_c + t_m - t_a$$
$$h = \frac{t_c + t_m - t_a}{t_m}$$

35.
$$L = \pi(r_1 + r_2) + 2x_1 + 2x_2$$
$$L = \pi r_1 + \pi r_2 + 2x_1 + 2x_2$$
$$\pi r_1 = L - \pi r_2 - 2x_1 - 2x_2$$
$$r_1 = \frac{L - \pi r_2 - 2x_1 - 2x_2}{\pi}$$

36.
$$I = \frac{VR_2 + VR_1(1+\mu)}{R_1R_2}$$
$$IR_1R_2 = VR_2 + VR_1 + VR_1\mu$$
$$VR_1\mu = IR_1R_2 - VR_2 + VR_1$$
$$\mu = \frac{IR_1R_2 - VR_2 + VR_1}{VR_1}$$

37.
$$P = \frac{V_1(V_2 - V_1)}{gJ}$$
$$gJP = V_1V_2 - V_1^2$$
$$V_1V_2 = V_1^2 + gJP$$
$$V_2 = \frac{V_1^2 + gJP}{V_1}$$

38.
$$W = T(S_1 - S_2) - Q$$
$$W + Q = TS_1 - TS_2$$
$$TS_2 = TS_1 - W - Q$$
$$S_2 = \frac{TS_1 - W - Q}{T}$$

39.
$$C = \frac{2eAk_1k_2}{d(k_1 + k_2)}$$
$$Cd(k_1 + k_2) = 2eAk_1k_2$$
$$e = \frac{Cd(k_1 + k_2)}{2Ak_1k_2}$$

40.

$$d = \frac{3LPx^2 - Px^3}{6EI}$$
$$6dEI = 3LPx^2 - Px^3$$
$$3LPx^2 = 6dEI + Px^3$$
$$L = \frac{6dEI + Px^3}{3Px^2}$$

41.

$$V = C\ \ 1 - \frac{n}{N}$$
$$V = C - \frac{Cn}{N}$$
$$V + \frac{Cn}{N} = C$$
$$\frac{Cn}{N} = C - V$$
$$Cn = CN - NV$$
$$n = \frac{CN - NV}{C}$$

42.

$$\frac{p}{P} = \frac{AI}{B + AI}$$
$$p(B + AI) = AIP$$
$$pB + AIp = AIP$$
$$pB = AIP - AIp$$
$$B = \frac{AIP - AIp}{p}$$

43.

$$p(C - n) + n = A$$
$$pC - pn + n = A$$
$$(-p + 1)n = A - pC$$
$$(1 - p)n = A - pC$$
$$n = \frac{A - pC}{1 - p}$$
$$n = \frac{13.0\text{ L} - 0.25(15.0\text{ L})}{1 - 0.25}$$
$$T_1 = \frac{13.0\text{ L} - 3.75\text{ L}}{0.75}$$
$$T_1 = \frac{9.25\text{ L}}{0.75}$$
$$T_1 = 12.333333\text{ L}$$
$$T_1 = 12\text{ L}$$

44. $P_t = P_c(1+0.500m^2)\dfrac{\pi}{2}$

$$P_c = \frac{P_t}{1+0.500m^2}$$

$$P_c = \frac{680\text{ W}}{1+0.500(0.925)^2}$$

$$P_c = \frac{680\text{ W}}{1+0.500(0.855625)}$$

$$P_c = \frac{680\text{ W}}{1+0.4278125}$$

$$P_c = \frac{680\text{ W}}{1.4278125}$$

$$P_c = 476.253009\text{ W}$$

$$P_c = 476\text{ W}$$

45. $F = \dfrac{9}{5}C + 32$

$$90.2 = \frac{9}{5}C + 32$$

$$\frac{5}{9}(90.2 - 32) = C$$

$$C = \frac{5}{9}\times 58.2$$

$$C = 32.3^\circ\text{C}$$

46. $V = \dfrac{1}{2}L(B+b)$

$$2V = BL + bL$$

$$bL = 2V - BL$$

$$b = \frac{2V - BL}{L}$$

$$b = \frac{2(38.6\text{ ft}^3) - (2.63\text{ ft}^2)(16.1\text{ ft})}{16.1\text{ ft}}$$

$$b = \frac{77.2\text{ ft}^3 - 42.343\text{ ft}^3}{16.1\text{ ft}}$$

$$b = \frac{34.857\text{ ft}^3}{16.1\text{ ft}}$$

$$b = 2.16503106\text{ ft}^2$$

$$b = 2.16\text{ ft}^2$$

47. $V_1 = \dfrac{VR_1}{R_1 + R_2}$

$V_1(R_1 + R_2) = VR_1$

$R_1 + R_2 = \dfrac{VR_1}{V_1}$

$R_2 = \dfrac{VR_1}{V_1} - R_1$

$R_2 = \dfrac{(12.0 \text{ V})(3.56\ \Omega)}{6.30 \text{ V}} - (3.56\ \Omega)$

$R_2 = 6.780952381\ \Omega - 3.56\ \Omega$

$R_2 = 3.220952381\ \Omega$

$R_2 = 3.22\ \Omega$

48. $\eta = \dfrac{1}{q + p(1-q)}$

$\eta\left[q + p(1-q)\right] = 1$

$\eta q + \eta p(1-q) = 1$

$\eta p(1-q) = 1 - \eta q$

$p = \dfrac{1 - \eta q}{\eta(1-q)}$

$p = \dfrac{1 - (0.66)(0.83)}{0.66(1 - 0.83)}$

$p = \dfrac{1 - 0.5478}{0.66(0.17)}$

$p = \dfrac{0.4522}{0.1122}$

$p = 4.03030303$

$p = 4$ processors

49. $d = v_2t_2 + v_1t_1$

$d = v_2(4 \text{ h}) + v_1(t + 2 \text{ h})$

$tv_1 + v_1(2 \text{ h}) = d - v_2(4 \text{ h})$

$tv_1 = d - v_2(4 \text{ h}) - v_1(2 \text{ h})$

$t = \dfrac{d - v_2(4 \text{ h}) - v_1(2 \text{ h})}{v_1}$

50. $C = x + 15y$

$15y = C - x$

$y = \dfrac{C - x}{15}$

1.12 Applied Word Problems

1. Let x = the number of 25 W lights.
Let $31-x$ = the number of 40 W lights.

$$25x+40(31-x)=1000$$
$$25x+1240-40x=1000$$
$$-15x=1000-1240$$
$$-15x=-240$$
$$x=16$$

There are 16 of the 25 W lights and(31– 16)=15 of the 40 W lights.
Check:

$$25\cdot 16+40(31-16)=1000$$
$$400+40(15)=1000$$
$$400+600=1000$$
$$1000=1000$$

2. Let$x$ = the number of slides with 5 mg.
Let $x-3$ = the number of slides with 6 mg.

$$(5\text{ mg})x=(6\text{ mg})(x-3)$$
$$(5\text{ mg})x=(6\text{ mg})x-18\text{ mg}$$
$$-x=-18$$
$$x=18\text{ slides}$$

There are 18 slides with 5 mg and (18−3)=15 slides with 6 mg.
Check:

$$5\text{ mg}(18)=6\text{ mg}(15)$$
$$90\text{ mg}=90\text{ mg}$$

3. Let t = the time for the shuttle to reach the satellite.

$$(29\ 500\text{ km/h})t=6000\text{ km}+(27\ 100\text{ km/h})t$$
$$(2400\text{ km/h})t=6000\text{ km}$$
$$t=\frac{6000\text{ km}}{2400\text{ km/h}}$$
$$t=2.500\text{ h}$$

It will take the shuttle 2.500 h to reach the satellite.
Check:

$$(29500\text{ km/h})(2.500\text{ h})=6000\text{ km}+(27100\text{ km/h})(2.500\text{ h})$$
$$73750\text{ km}=6000\text{ km}+67750\text{ km}$$
$$73750\text{ km}=73750\text{ km}$$

4. Let x = the number of litres of 50% methanol blend that must be added.

$$0.0600(7250\text{ L})+0.500(x)=0.100(7250\text{ L}+x)$$
$$435\text{ L}+0.500(x)=725\text{ L}+0.100x$$
$$0.400(x)=290\text{ L}$$
$$x=\frac{290\text{ L}}{0.400}$$
$$x=725\text{ L}$$

725 L of the 50% methanol blend must be added.

Check:

$$\begin{aligned} 0.0600(7250\text{ L}) + 0.500(725\text{ L}) &= 0.100(7250\text{ L} + 725\text{ L}) \\ 435\text{ L} + 362.5\text{ L} &= 0.1(7975\text{ L}) \\ 797.5\text{ L} &= 797.5\text{ L} \end{aligned}$$

5. Let x = the cost of the car 6 years ago.
Let x + \$5000 = the cost of the car model today.

$$\begin{aligned} x + (x + \$5000) &= \$49\ 000 \\ 2x &= \$44\ 000 \\ x &= \frac{\$44\ 000}{2} \\ x &= \$22\ 000 \end{aligned}$$

The cost of the car 6 years ago was \$22 000, and the cost of the today's model is (\$22 000 + 5000) = \$27 000.
Check:

$$\begin{aligned} \$22\ 000 + (\$22\ 000 + \$5000) &= \$49\ 000 \\ \$22\ 000 + \$27\ 000 &= \$49\ 000 \\ \$49\ 000 &= \$49\ 000 \end{aligned}$$

6. Let x = the flow from the first stream in m^3/s.
Let x – 1700 ft^3/s = the flow from the second streamin m^3/s.

$$\begin{aligned} x + (x - 1700\text{ ft}^3/\text{s}) &= \frac{1.98\times10^7\text{ ft}^3}{3600\text{ s}} \\ 2x - 1700\text{ ft}^3/\text{s} &= 5500\text{ ft}^3/\text{s} \\ 2x &= 7200\text{ ft}^3/\text{s} \\ x &= \frac{7200\text{ ft}^3/\text{s}}{2} \\ x &= 3600\text{ ft}^3/\text{s} \end{aligned}$$

The first stream flows $3600\text{ ft}^3/\text{s}$ and the second stream flows 3600 ft^3/s – 1700 ft^3/s = 1900 ft^3/s.
Check:

$$\begin{aligned} 3600\text{ ft}^3/\text{s} + (3600\text{ ft}^3/\text{s} - 1700\text{ ft}^3/\text{s}) &= \frac{1.98\times10^7\text{ ft}^3}{3600\text{ s}} \\ 7200\text{ ft}^3/\text{s} - 1700\text{ ft}^3/\text{s} &= 5500\text{ ft}^3/\text{s} \\ 5500\text{ ft}^3/\text{s} &= 5500\text{ ft}^3/\text{s} \end{aligned}$$

7. Let x = the number of cars recycled the first year.
Let x+500000 = the number of cars recycled the second year.

$$\begin{aligned} x + (x + 500000\text{ cars}) &= 6900000\text{ cars} \\ 2x + 500000\text{ cars} &= 6900000\text{ cars} \\ 2x &= 6400000\text{ cars} \\ x &= \frac{6400000\text{ cars}}{2} \\ x &= 3200000\text{ cars} \end{aligned}$$

The first year,3.2×10^6 cars were recycled, and the second year (3200000 + 500000)= 3.7×10^6 cars were recycled.
Check:

$$\begin{aligned} 3200000\text{ cars} + (3200000\text{ cars} + 500000\text{ cars}) &= 6900000\text{ cars} \\ 3200000\text{ cars} + 3700000\text{ cars} &= 6900000\text{ cars} \\ 6900000\text{ cars} &= 6900000\text{ cars} \end{aligned}$$

8. Let x = the number of hits to the website on the first day.
Let 1/2 x = the number of hits on the second day.

$$x + 1/2\,x = 495000 \text{ hits}$$
$$3/2\,x = 495000 \text{ hits}$$
$$x = \frac{495000 \text{ hits}}{3/2}$$
$$x = 330000 \text{ hits}$$

The first day there were 330000 hits, the second day there were 1/2(33000 hits= 165000 hits.
Check:

$$330000 \text{ hits} + 1/2(330000 \text{ hits}) = 495000 \text{ hits}$$
$$330000 \text{ hits} + 165000 \text{ hits} = 495000 \text{ hits}$$
$$495000 \text{ hits} = 495000 \text{ hits}$$

9. Let x = the number acres of land leased for \$200 per acre.
Let 140 – x = the number of acres of land leased for \$300 per acre.

$$\$200 / \text{acre}\; x + \$300 / \text{acre}(140 \text{ acre} - x) = \$37\ 000$$
$$-\$100 / \text{acre}\ (x) = -\$5\ 000$$
$$x = \frac{-\$5000}{-\$100 / \text{acre}}$$
$$x = 50 \text{ acres}$$

There are 50 acres leased at \$200 per acre and (140 acres – 50 acres) = 90 hectares leased for \$300 per hectare.
Check:

$$\$200 / \text{acre}\ (50 \text{ acres}) + \$300 / \text{acre}(140 \text{ acres} - 50 \text{ acres}) = \$37\ 000$$
$$\$10\ 000 + \$27\ 000 = \$37\ 000$$
$$\$37\ 000 = \$37\ 000$$

10. Let x = the first dose in mg.
Let x + 660 mg = the second dose in mg.

$$x + x + 660 \text{ mg} = 2000 \text{ mg}$$
$$2x = 1340 \text{ mg}$$
$$x = \frac{1340 \text{ mg}}{2}$$
$$x = 670 \text{ mg}$$

The first dose is 670 mg, and the second dose is (670 mg + 660 mg) = 1130 mg.
Check:

$$670 \text{ mg} + 670 \text{ mg} + 660 \text{ mg} = 2000 \text{ mg}$$
$$670 \text{ mg} + 1330 \text{ mg} = 2000 \text{ mg}$$
$$2000 \text{ mg} = 2000 \text{ mg}$$

11. Let x = the amount of pollutant after modification in ppm/h.

$$(5 \text{ h})x = (3 \text{ h})150 \text{ ppm/h}$$
$$x = \frac{450 \text{ ppm}}{5 \text{ h}}$$
$$x = 90 \text{ ppm/h}$$

The amount of pollutant after modification is 90 ppm/h. The device reduced emissions by (150 ppm/h – 90 ppm/h) = 60 ppm/h.
Check:

$$(5 \text{ h})90 \text{ ppm/h} = (3 \text{ h})150 \text{ ppm/h}$$
$$450 \text{ ppm} = 450 \text{ ppm}$$

12. Let $x - 13$ = the number of teeth that the first meshed spur has.
Let x = the number of teeth that the second meshed spur has.
Let $x + 15$ = the number of teeth that the third meshed spur has.

$$\begin{aligned} x - 13 \text{ teeth} + x + x + 15 \text{ teeth} &= 107 \text{ teeth} \\ 3x + 2 &= 107 \text{ teeth} \\ 3x &= 105 \text{ teeth} \\ x &= \frac{105 \text{ teeth}}{3} \\ x &= 35 \text{ teeth} \end{aligned}$$

The first spur has (35 – 13)= 22 teeth, the second spur has 35 teeth, and the third spur has (35 + 15) = 50 teeth.
Check:

$$\begin{aligned} 35 \text{ teeth} - 13 \text{ teeth} + 35 \text{ teeth} + 35 \text{ teeth} + 15 \text{ teeth} &= 107 \text{ teeth} \\ 107 \text{ teeth} &= 107 \text{ teeth} \end{aligned}$$

13. Let x = amount paid per month for first six months.
Let $x + 10$ = amount paid per month for final four months.

$$\begin{aligned} (6 \text{ mo})x + (4 \text{ mo})(x + \$10/\text{mo}) &= \$890 \\ (10 \text{ mo})x + \$40 &= \$890 \\ (10 \text{ mo})x &= \$850 \\ x &= \frac{\$850}{10 \text{ mo}} \\ x &= \$85/\text{mo} \end{aligned}$$

The bill was \$85/mo for the first six months and \$95/mo for the next four months.
Check:

$$\begin{aligned} (6 \text{ mo})\$85/\text{mo} + (4 \text{ mo})(\$85/\text{mo} + \$10/\text{mo}) &= \$890 \\ \$510 + (4 \text{ mo})(\$95/\text{mo}) &= \$890 \\ \$510 + \$380 &= \$890 \\ \$890 &= \$890 \end{aligned}$$

14. Let x = amount paid per month for first year.
Let $x + 30$ = amount paid per month for next two years.
Let $(x + 30)+20= x + 50$= amount paid per month for final two years.

$$\begin{aligned} (12 \text{ mo})x + (24 \text{ mo})(x + \$30/\text{mo}) + (24 \text{ mo})(x + \$50/\text{mo}) &= \$7320 \\ (12 \text{ mo})x + (24 \text{ mo})x + \$720 + (24 \text{ mo})x + \$1200 &= \$7320 \\ (60 \text{ mo})x + \$1920 &= \$7320 \\ (60 \text{ mo})x &= \$5400 \\ x &= \frac{\$5400}{60 \text{ mo}} \\ x &= \$90/\text{mo} \end{aligned}$$

For the first year, the bill was \$90/mo, during years 2 and 3, the bill was \$120/mo, and during years 4 and 5, the bill was \$140/mo.
Check:

$$\begin{aligned} (12 \text{ mo})(\$90/\text{mo}) + (24 \text{ mo})(\$90/\text{mo} + \$30/\text{mo}) + (24 \text{ mo})(\$90/\text{mo} + \$50/\text{mo}) &= \$7320 \\ \$1080 + (24 \text{ mo})(\$120/\text{mo}) + (24 \text{ mo})(\$140/\text{mo}) &= \$7320 \\ \$1080 + \$2880 + \$3360 &= \$7320 \\ \$7320 &= \$7320 \end{aligned}$$

15. Let x = the first current in μA.
Let $2x$ = the second current in μA.
Let $x + 9.2\ \mu$A = the third current in μA

$$x+2x+x+9.2\ \mu\text{A}=0\ \mu\text{A}$$
$$4x=-9.2\ \mu\text{A}$$
$$x=\frac{-9.2\ \mu\text{A}}{4}$$
$$x=-2.3\ \mu\text{A}$$

The first current is $-2.3\ \mu$A, the second current is $2(-2.3\ \mu\text{A}) = -4.6\ \mu$A, and the third current is $(-2.3\ \mu\text{A} + 9.2\ \mu\text{A}) = 6.9\ \mu$A.

Check:

$$-2.3\ \mu\text{A}+2(-2.3\ \mu\text{A})+(-2.3)\ \mu\text{A}+9.2\ \mu\text{A}=0\ \mu\text{A}$$
$$-2.3\ \mu\text{A}-4.6\ \mu\text{A}-2.3\mu\text{A}+9.2\ \mu\text{A}=0\ \mu\text{A}$$
$$0\ \mu\text{A}=0\ \mu\text{A}$$

16. Let x = the number of trucks in the first fleet.
Let $x + 5$ = the number of trucks in the second fleet.

$$(8\text{ h})x+(6\text{ h})(x+5)=198\text{ h}$$
$$(8\text{ h})x+(6\text{ h})x+30\text{ h}=198\text{ h}$$
$$(14\text{ h})x=168\text{ h}$$
$$x=\frac{168\text{ h}}{14\text{ h}}$$
$$x=12\text{ trucks}$$

There are 12 trucks in the first fleet and (12 trucks + 5 trucks) = 17 trucks in the second fleet.

Check:

$$(8\text{ h})(12)+(6\text{ h})(12+5)=198\text{ h}$$
$$96\text{ h}+(6\text{ h})(17)=198\text{ h}$$
$$96\text{ h}+102\text{ h}=198\text{ h}$$
$$198\text{ h}=198\text{ h}$$

17. Let x = the length of the first pipeline in km.
Let $x + 2.6$ km = the length of the 3 other pipelines.

$$x+3(x+2.6\text{ km})=35.4\text{ km}$$
$$x+3x+7.8\text{ km}=35.4\text{ km}$$
$$4x=27.6\text{ km}$$
$$x=\frac{27.6\text{ km}}{4}$$
$$x=6.9\text{ km}$$

The first pipeline is 6.9 km long, and the other three pipelines are each (6.9 km + 2.6 km) = 9.5 km long.

Check:

$$6.9\text{ km}+3(6.9\text{ km}+2.6\text{ km})=35.4\text{ km}$$
$$6.9\text{ km}+3(9.5\text{ km})=35.4\text{ km}$$
$$6.9\text{ km}+28.5\text{ km}=35.4\text{ km}$$
$$35.4\text{ km}=35.4\text{ km}$$

18. Let x = the power of the first generator in MW.
Let 750 MW−x = the power of the second generator in MW.

$$0.65x + 0.75(750\text{ MW} - x) = 530\text{ MW}$$
$$0.65x + 562.5\text{ MW} - 0.75x = 530\text{ MW}$$
$$-0.1x = -32.5\text{ MW}$$
$$x = \frac{-32.5\text{ MW}}{-0.1}$$
$$x = 325\text{ MW}$$

The first generator produces 325 MW of power, and the second generator produces (750 MW – 325 MW) = 425 MW of power.
Check:

$$0.65(325\text{ MW}) + 0.75(750\text{ MW} - (325\text{ MW})) = 530\text{ MW}$$
$$211.25\text{ MW} + 0.75(425\text{ MW}) = 530\text{ MW}$$
$$211.25\text{ MW} + 318.75\text{ MW} = 530\text{ MW}$$
$$530\text{ MW} = 530\text{ MW}$$

19. Let x = the number of deluxe systems.
Let $2x$ = the number economy systems.
Let x+75= the number of econo-plus systems.

$$\$140x + \$40(2x) + \$80(x + 75) = \$42000$$
$$\$140x + \$80x + \$80x + \$6000 = \$42000$$
$$\$300x = \$36000$$
$$x = 120\text{ systems}$$

There are 120 deluxe systems, 240 economy systems, and 195 econo-plus systems sold.
Check:

$$\$140(120) + \$40(240) + \$80(195) = \$42000$$
$$\$16800 + \$9600 + \$15600 = \$42000$$
$$\$42000 = \$42000$$

20. The amount of lottery winnings after taxes is \$20 000 ×(1−0.25) = \$15 000.
Let x = the amount of money invested at a 40% gain.
Let \$15 000− x = the amount of money invested at a 10% loss.

$$0.40x - 0.10(\$15\ 000 - x) = \$2000$$
$$0.40x - \$1500 + 0.10x = \$2000$$
$$0.50x = \$3500$$
$$x = \frac{\$3500}{0.50}$$
$$x = \$7000$$

The 40% gain investment had \$7000 invested, and the 10% loss investment had (\$15 000− \$7000) = \$8000 invested.
Check:

$$0.40(\$7000) - 0.10(\$15\ 000 - \$7000) = \$2000$$
$$\$2800 - \$1500 + 0.10(\$7000) = \$2000$$
$$\$2800 - \$1500 + \$700 = \$2000$$
$$\$2000 = \$2000$$

21. Let x = the amount of time in seconds between when the start of the trains pass each other to when the end of the trains pass each other.
The total distance the ends must travel in this time is 960 feet. We first convert mi/hr into ft/sec.

$$1\text{ mi/hr } = \frac{5280\text{ ft}}{3600\text{ s}} = \frac{22\text{ ft}}{15\text{ s}} = \frac{22}{15}\text{ ft/s}$$

Therefore, train A travels at 60(22/15)=88 ft/s and train B travels at 40(22/15)=176/3 ft/s.

$$(88\text{ ft/s})x + (176/3\text{ ft/s})x = 960\text{ ft}$$
$$(440/3\text{ ft/s})x = 960\text{ ft}$$
$$x = \frac{960\text{ ft}}{440/3\text{ ft/s}}$$
$$x = \frac{72}{11}\text{ s}$$

The trains completely pass each other in about 6.55 seconds.
Check:

$$(88\text{ ft/s})\,\frac{72}{11}\text{ s} + (176/3\text{ ft/s})\frac{72}{11}\text{ s} = 960\text{ ft}$$
$$576\text{ ft} + 384\text{ ft} = 960\text{ ft}$$
$$960\text{ ft} = 960\text{ ft}$$

22. Let x = the mortgage payment and $x/0.23$=the monthly income.

$$x/0.23 - x = \$3850$$
$$x\left(\frac{1}{0.23} - 1\right) = \$3850$$
$$x\left(\frac{1}{0.23} - \frac{0.23}{0.23}\right) = \$3850$$
$$x\left(\frac{0.77}{0.23}\right) = \$3850$$
$$x = \$3850 \cdot \frac{0.23}{0.77}$$
$$x = \$1150$$

The mortgage payment is \$1150 and the monthly income is \$5000.
Check:

$$\$1150/0.23 - \$1150 = \$3850$$
$$\$5000 - \$1150 = \$3850$$
$$\$3850 = \$3850$$

23. Let x = the amount of time the skier spends on the ski lift in minutes.
Let 24 minutes $-x$ = the amount of time the skier spends skiing down the hill in minutes.

$$(50\text{ m/min})x = (150\text{ m/min})(24\text{ min} - x)$$
$$(50\text{ m/min})x = 3600\text{ m} - (150\text{ m/min})x$$
$$(200\text{ m/min})x = 3600\text{ m}$$
$$x = \frac{3600\text{ m}}{200\text{ m/min}}$$
$$x = 18\text{ min}$$

The length of the slope is 18 minutes × 50 m/minute = 900m.

Check:

$$\begin{aligned}(50\text{ m/min})18\text{ min} &= (150\text{ m/min})(24\text{ min} - 18\text{ min})\\ 900\text{ m} &= 3600\text{ m} - (150\text{ m/min})(18\text{ min})\\ 900\text{ m} &= 3600\text{ m} - 2700\text{ m}\\ 900\text{ m} &= 900\text{ m}\end{aligned}$$

24. Let x = the speed of sound.
Let x – 120 mi/h = speed travelled for 1 h.
Let x + 410 mi/h = the speed travelled for 3 h.

$$\begin{aligned}1\text{ h}(x - 120\text{ mi/h}) + 3\text{ h}(x + 410\text{ mi/h}) &= 3990\text{ mi}\\ (1\text{ h})x - (1\text{ h})(120\text{ mi/h}) + (3\text{ h})x + (3\text{ h})(410\text{ mi/h}) &= 3990\text{ mi}\\ (1\text{ h})x - 120\text{ mi} + (3\text{ h})x + 1230\text{ mi} &= 3990\text{ mi}\\ (4\text{ h})x &= 2880\text{ mi}\\ x &= \frac{2880\text{ mi}}{4\text{ h}}\\ x &= 720\text{ mi/h}\end{aligned}$$

The speed of sound is 720 mi/h.
Check:

$$\begin{aligned}1\text{ h}(720\text{ mi/h} - 120\text{ mi/h}) + 3\text{ h}(720\text{ mi/h} + 410\text{ mi/h}) &= 3990\text{ mi}\\ (1\text{ h})(600\text{ mi/h}) + (3\text{ h})(1130\text{ mi/h}) &= 3990\text{ mi}\\ 600\text{ mi} + 3390\text{ mi} &= 3990\text{ mi}\\ 3990\text{ mi} &= 3990\text{ mi}\end{aligned}$$

25. Let x = the speed the train leaving England in km/h.
Let x + 8 km/h = speed of the train leaving France in km/h.
The distance travelled by each train is speed ×time.

$$\begin{aligned}x\left(\frac{17\text{ min}}{60\text{ min/ h}}\right) + (x + 8\text{ km/h})\left(\frac{17\text{ min}}{60\text{ min/ h}}\right) &= 50\text{ km}\\ (0.28333\text{ h})x + (x + 8\text{ km/h})(0.28333\text{ h}) &= 50\text{ km}\\ (0.28333\text{ h})x + (0.28333\text{ h})x + 2.26667\text{ km} &= 50\text{ km}\\ (0.56666\text{ h})x &= 47.73333\text{ km}\\ x &= \frac{47.73333\text{ km}}{0.56666\text{ h}}\\ x &= 84.23529421\text{ km/h}\\ x &= 84.2\text{ km/h}\end{aligned}$$

The train leaving England was travelling at 84.2 km/h, and the train leaving France was travelling at (84.2 km/h + 8 km/h) = 92.2 km/h.
Check:

$$\begin{aligned}84.23529421\text{ km/h}\left(\frac{17\text{ min}}{60\text{ min/ h}}\right) + (84.23529421\text{ km/h} + 8\text{ km/h})\left(\frac{17\text{ min}}{60\text{ min/ h}}\right) &= 50\text{ km}\\ 23.86666\text{ km} + (92.23529421\text{ km/h})\left(\frac{17\text{ min}}{60\text{ min/ h}}\right) &= 50\text{ km}\\ 23.86666\text{ km} + 26.13333\text{ km} &= 50\text{ km}\\ 50\text{ km} &= 50\text{ km}\end{aligned}$$

26. Let x = time left until the appointment.
Let x– 10.0 min = time taken to get to the appointment travelling at 60.0 mi/h.
Let x– 5.0 min = time taken to get to the appointment travelling at 45.0 mi/h.
The distance travelled by the executive in each scenario is the same. Distance = speed × time

$$60.0\text{ mi/h}\left(x-\frac{10.0\text{ min}}{60\text{ min/h}}\right)=45\text{ mi/h}\left(x-\frac{5.0\text{ min}}{60\text{ min/h}}\right)$$
$$(60.0\text{ mi/h})x-60\text{ mi/h}\left(\frac{10.0\text{ min}}{60\text{ min/h}}\right)=(45\text{ mi/h})x-45\text{ mi/h}\left(\frac{5.0\text{ min}}{60\text{ min/h}}\right)$$
$$(60.0\text{ mi/h})x-10\text{ mi}=(45.0\text{ mi/h})x-3.75\text{ mi}$$
$$(15.0\text{ mi/h})x=6.25\text{ mi}$$
$$x=\frac{6.25\text{ mi}}{15.0\text{ mi/h}}$$
$$x=0.416666667\text{ h}$$
$$x=0.416666667\text{ h x }60\text{ min/h}$$
$$x=\ 25\text{ min}$$

There is 25 minutes left until the executive's appointment.
Check:

$$60.0\text{ mi/h}\left(0.41667\text{ h}-\frac{10.0\text{ min}}{60\text{ min/h}}\right)=45\text{ mi/h}\left(0.41667\text{ h}-\frac{5.0\text{ min}}{60\text{ min/h}}\right)$$
$$60.0\text{ mi/h}(0.25\text{ h})=45\text{ mi/h}(0.33333\text{ h})$$
$$15\text{ mi}=15\text{ mi}$$

27. Let x – 30.0 s = time since the first car started moving in the race in seconds.
Let x= time since the second car started the race in seconds.
The distance travelled by each car will be the same at the point where the first car overtakes the second car.
Distance = speed × time.

$$260.0\text{ ft/s}(x-30.0\text{ s})=240.0\text{ ft/s}(x)$$
$$(260.0\text{ ft/s})x-(260.0\text{ ft/s})(30.0\text{ s})=(240.0\text{ ft/s})x$$
$$(260.0\text{ ft/s})x-7800\text{ ft}=(240.0\text{ ft/s})x$$
$$(20.0\text{ ft/s})x=7800\text{ ft}$$
$$x=\frac{7800\text{ ft}}{20.0\text{ ft/s}}$$
$$x=390\text{ s}$$

The first car will overtake the second car after 390 s. The first car travels 260 ft/s × (390 s – 30 s) = 93600 ft by this point. 8 laps around the track is 2.5 mi/lap. 8 laps × 5280 ft/mi = 105,600 ft, so the first car will already be in the lead at the end of the 8th lap.
Check:

$$260.0\text{ ft/s}(390.0\text{ s}-30.0\text{ s})=240.0\text{ ft/s}(390\text{ s})$$
$$260.0\text{ ft/s}(360.0\text{ s})=240.0\text{ ft/s}(390\text{ s})$$
$$93,600\text{ ft}=93,600\text{ ft}$$

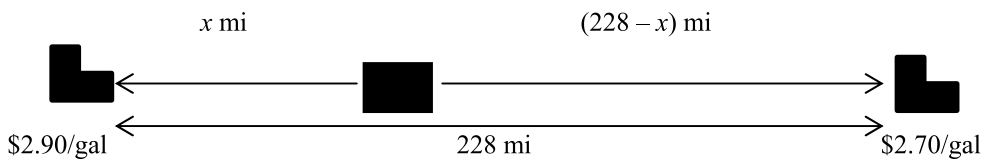
28. Let x = the number of the first chips that is defective 0.50%.
Let 6100 $-x$ = the number of the second chips that is defective 0.80%.

$$0.0050(x)+0.0080(6100\text{ chips}-x)=38\text{ chips}$$
$$(0.0050)x+48.8\text{ chips}-(0.0080)x=38\text{ chips}$$
$$-(0.0030)x=-10.8\text{ chips}$$
$$x=\frac{-10.8\text{ chips}}{-0.0030}$$
$$x=3600\text{ chips}$$

There are 3600 chips that are 0.50% defective and (6100 chips – 3600 chips) = 2500 chips that are defective 0.80%.
Check:

$$0.0050(3600\text{ chips})+0.0080(6100\text{ chips}-3600\text{ chips})=38\text{ chips}$$
$$18\text{ chips}+0.0080(2500\text{ chips})=38\text{ chips}$$
$$18\text{ chips}+20\text{ chips}=38\text{ chips}$$
$$38\text{ chips}=38\text{ chips}$$

29.



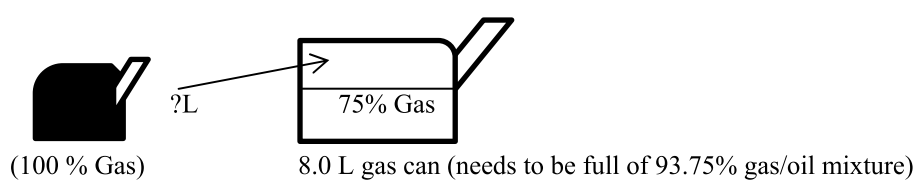
Assuming that the customer is located between the two gasoline distributors:
Let x = the distance in km to the first gasoline distributor that costs \$2.90/gal.
Let 228 mi$-$ x= the distance in km to the second gasoline distributor that costs \$2.70/gal.

$$\$2.90+\$0.002(x)=\$2.70+\$0.002(228-x)$$
$$\$2.90+\$0.002(x)=\$2.70+\$0.456-\$0.002(x)$$
$$\$0.004(x)=\$0.256$$
$$x=\frac{\$0.256}{\$0.004}$$
$$x=64\text{ mi}$$

The customer is 64 mi away from the first gas distributor (\$2.90/gal) and (228 mi – 64 mi)= 164 mi away from the second gas distributor (\$2.70).
Check:

$$\$2.90+\$0.002(64)=\$2.70+\$0.002(228-64)$$
$$\$2.90+\$0.128=\$2.70+\$0.002(164)$$
$$\$3.028=\$2.70+\$0.328$$
$$\$3.028=\$3.028$$

30.



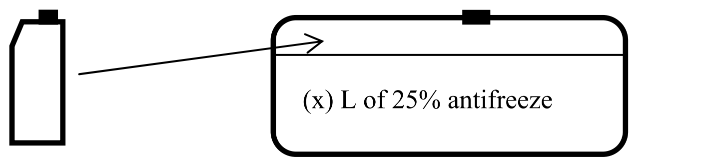
A 15:1 gas/oil mixture is 15/16 gasoline = 93.75%.
Let x = the amount of 100% gasoline added in L.
Let 8.0 L – x = the amount of 75% gasoline mixture in L.

$$1.00(x)+0.75(8.0\text{ L}-x)=0.9375(8.0\text{ L})$$
$$1.00(x)+6.0\text{ L}-0.75(x)=7.5\text{ L}$$
$$0.25(x)=1.5\text{ L}$$
$$x=\frac{1.5\text{ L}}{0.25}$$
$$x=6.0\text{ L}$$

6.0 L of 100% gasoline must be added to the 75% gas/oil mixture to make 8 L of 15:1 gasoline/ oil.
Check:

$$1.00(6.0\text{ L})+0.75(8.0\text{ L}-6.0\text{ L})=0.9375(8.0\text{ L})$$
$$6\text{ L}+0.75(2.0\text{ L})=7.5\text{ L}$$
$$6\text{ L}+1.5\text{ L}=7.5\text{ L}$$
$$7.5\text{ L}=7.5\text{ L}$$

31.



100% Antifreeze 12.0 L radiator (needs to be filled with 50% mixture)

Let x = the amount in L of 25% antifreeze left in radiator
Let 12.0 L – x = the amount of 100% antifreeze added in L.

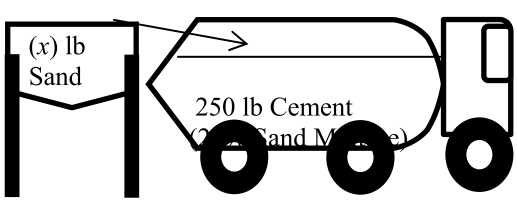
$$0.25(x)+1.00(12.0\text{ L}-x)=0.5(12.0\text{ L})$$
$$0.25(x)+12.0\text{ L}-1.00(x)=6.0\text{ L}$$
$$-0.75(x)=-6.0\text{ L}$$
$$x=\frac{-6.0\text{ L}}{-0.75}$$
$$x=8.0\text{ L}$$

There needs to be 8L of 25% antifreeze left in radiator, so (12.0 L – 8.0 L) = 4.0 L must be drained.

Check:

$$0.25(8.0\text{ L})+1.00(12.0\text{ L}-8.0\text{ L})=0.5(12.0\text{ L})$$
$$2\text{ .0L}+1.00(4.0\text{ L})=6.0\text{ L}$$
$$2.0\text{ L}+4.0\text{ L}=6.0\text{ L}$$
$$6.0\text{ L}=6.0\text{ L}$$

32.



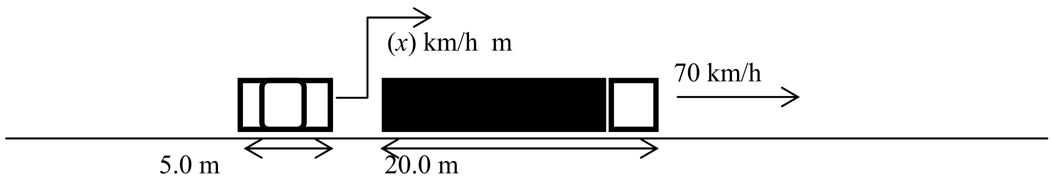
Let x = the amount of sand added.
Let 250 lb + x = the amount in lb of the final 25% sand mixture.

$$1.00(x) + 0.22(250 \text{ lb}) = 0.25(250 \text{ lb} + x)$$
$$1.00(x) + 55 \text{ lb} = 62.5 \text{ lb} + 0.25(x)$$
$$0.75(x) = 7.5 \text{ lb}$$
$$x = \frac{7.5 \text{ lb}}{0.75}$$
$$x = 10 \text{ lb}$$

Check:

$$1.00(10 \text{ lb}) + 0.22(250 \text{ lb}) = 0.25(250 \text{ lb} + 10 \text{ lb})$$
$$10 \text{ lb} + 55 \text{ lb} = 62.5 \text{ lb} + 2.5 \text{ lb}$$
$$65 \text{ lb} = 65 \text{ lb}$$

33.



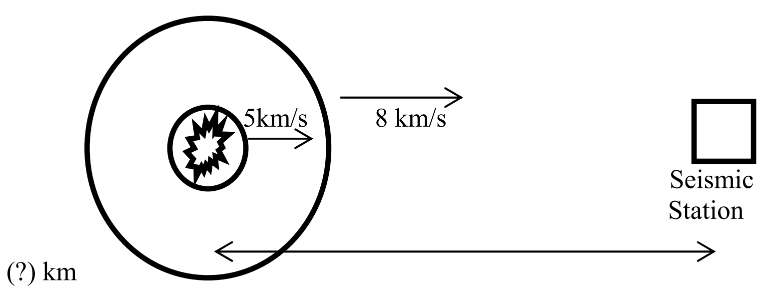
Let x = the speed the car needs to travel in km/h to pass the semi in 10 s.
Speed = distance/time. 10 s is 10s/3600 s/h = 0.002777777 h.

$$x = \frac{\text{distance needed to pass truck + distance travelled by truck in 10s}}{10s}$$
$$x = \frac{0.025 \text{ km} + 70 \text{ km/h}\left(0.0027777 \text{ h}\right)}{0.0027777 \text{ h}}$$
$$x = \frac{0.025 \text{ km} + 0.19444 \text{ km}}{0.0027777 \text{ h}}$$
$$x = \frac{2.19444 \text{ km}}{0.0027777 \text{ h}}$$
$$x = 79 \text{ km/h}$$

The car needs to travel at a speed of 79 km/h to pass the semitrailer in 10s.
Check:

$$79 \text{ km/h} = \frac{0.025 \text{ km} + 70 \text{ km/h}\left(0.0027777 \text{ h}\right)}{0.0027777 \text{ h}}$$
$$79 \text{ km/h} = \frac{0.025 \text{ km} + 0.19444 \text{ km}}{0.0027777 \text{ h}}$$
$$79 \text{ km/h} = 79 \text{ km/h}$$

34.



Let x = the time the first wave takes to travel to the seismic station in s.
Let x + 120 s = the time the first wave takes to travel to the seismic station in s.
Distance =speed × time. The distances travelled by both waves to the seismic station are the same. 2.0 min is (2.0 min × 60 s/min) = 120 s.

$$8.0 \text{ km/s}(x) = 5.0 \text{ km/s}(x + 120 \text{ s})$$
$$8.0 \text{ km/s}(x) = 5.0 \text{ km/s}(x) + (5 \text{ km/s})(120 \text{ s})$$
$$3.0 \text{ km/s}(x) = 600 \text{ km}$$
$$x = \frac{600 \text{ km}}{3.0 \text{ km/s}}$$
$$x = 200 \text{ s}$$

The distance to the seismic station is (200 s × 8.0 km/s) = 1600 km.
Check:

$$8.0 \text{ km/s}(200 \text{ s}) = 5.0 \text{ km/s}(200 \text{ s} + 120 \text{ s})$$
$$1600 \text{ km} = 5.0 \text{ km/s}(320 \text{ s})$$
$$1600 \text{ km} = 1600 \text{ km}$$

Review Exercises

1. False, because $|0| = 0$ which is not a positive value.

2. True. The order of operations dictates performing the division first, then the subtraction.

3. False. The reported answer should have only two significant digits.

4. False. Had the problem been given as $(2a)^3 = 8a^3$, then it would be true.

5. True.

6. False. The left-hand side, $-\sqrt{-4}$, is not a real number, in fact.

7. False. The left-hand side simplifies to $4x - (2x + 3) = 4x - 2x - 3 = 2x - 3$.

8. True.

9. False. The left-hand side simplifies to $\frac{6x+2}{2} = \frac{6x}{2} + \frac{2}{2} = 3x + 1$.

10. True.

11. False. Solving for c yields

$$a - bc = d$$
$$-bc = d - a$$
$$c = \frac{d - a}{-b}$$
$$c = -\frac{d - a}{b}$$

12. False. It is likely that one should set up a phrase such as 'let x be the number of gears of the first type…'

13. $(-2) + (-5) - 3 = -2 - 5 - 3 = -10$

14. $6-8-(-4)=6-8+4=2$

15. $\dfrac{(-5)(6)(-4)}{(-2)(3)}=\dfrac{(20)\cancel{(6)}}{-\cancel{(6)}}=-20$

16. $\dfrac{(-9)(-12)(-4)}{24}=\dfrac{108(-4)}{24}=\dfrac{-432}{24}=-18$

17. $-5-|2(-6)|+\dfrac{-15}{3}=-5-|-12|+(-5)=-5-12-5=-22$

18. $3-5|-3-2|-\dfrac{12}{-4}=3-5|-5|-(-3)=3-5(5)+3=6-25=-19$

19. $\dfrac{18}{3-5}-(-4)^2=\dfrac{18}{-2}-(-4)(-4)=-9-16=-25$

20. $-(-3)^2-\dfrac{-8}{(-2)-|-4|}=-(-3)(-3)-\dfrac{-8}{(-2)-4}=-9-\dfrac{-8}{-6}=-\dfrac{27}{3}-\dfrac{4}{3}=-\dfrac{31}{3}$

21. $\sqrt{16}-\sqrt{64}=\sqrt{(4)(4)}-\sqrt{(8)(8)}=4-8=-4$

22. $-\sqrt{81+144}=-\sqrt{225}=-\sqrt{(5)(5)(3)(3)}=-(3)(5)=-15$

23. $(\sqrt{7})^2-\sqrt[3]{8}=(\sqrt{7})(\sqrt{7})-\sqrt[3]{(2)(2)(2)}=7-2=5$

24. $-\sqrt[4]{16}+(\sqrt{6})^2=-\sqrt[4]{(2)(2)(2)(2)}+(\sqrt{6})(\sqrt{6})=-2+6=4$

25. $(-2rt^2)^2=(-2)^2r^2t^{2x2}=4r^2t^4$

26. $(3a^0b^{-2})^3=(3)^3(1)^3b^{-2\times 3}=27(1)b^{-6}=\dfrac{27}{b^6}$

27. $-3mn^{-5}t(8m^{-3}n^4)=-(3)(8)m^{1-3}n^{-5+4}t=-24m^{-2}n^{-1}t=-\dfrac{24t}{m^2n}$

28. $\dfrac{15p^4q^2r}{5pq^5r}=\dfrac{3p^{4-1}\cancel{r}}{q^{5-2}\cancel{r}}=\dfrac{3p^3}{q^3}$

29. $\dfrac{-16N^{-2}(NT^2)}{-2N^0T^{-1}}=\dfrac{8N^{-2+1}T^{2+1}}{(1)}=\dfrac{8N^{-1}T^3}{(1)}=\dfrac{8T^3}{N}$

30. $\dfrac{-35x^{-1}y(x^2y)}{5xy^{-1}}=\dfrac{-7y^{1+1+1}x^2}{x^{1+1}}=\dfrac{-7y^3\cancel{x^2}}{\cancel{x^2}}=-7y^3$

31. $\sqrt{45}=\sqrt{(5)(3)(3)}=3\sqrt{5}$

32. $\sqrt{9+36}=\sqrt{45}=\sqrt{(5)(3)(3)}=3\sqrt{5}$

33. 8000 has 1 significant digit. Rounded to 2 significant digits, it is $8\bar{0}00$.

34. 21490 has 4 significant digits. Rounded to 2 significant digits, it is 21000.

35. 9.050 has 4 significant digits. Rounded to 2 significant digits, it is 9.0.

36. 0.7000 has 4 significant digits. Rounded to 2 significant digits, it is 0.70.

37.
$$\begin{aligned}37.3-16.92(1.067)^2 &= 37.3-16.92(1.138489)\\ &= 37.3-19.26323388\\ &= 18.03676612\end{aligned}$$
which rounds to 18.0.

38.
$$\begin{aligned}\frac{8.896\times10^{-12}}{-3.5954-6.0449} &= \frac{8.896\times10^{-12}}{-9.6403}\\ &= -9.227928591\times10^{-13}\end{aligned}$$
which rounds to -9.228×10^{-13}.

39.
$$\begin{aligned}\frac{\sqrt{0.1958+2.844}}{3.142(65)^2} &= \frac{\sqrt{3.0398}}{3.142(4225)}\\ &= \frac{1.743502223}{13274.95}\\ &= 0.000131337\end{aligned}$$
which rounds to 1.3×10^{-4}.

40.
$$\begin{aligned}\frac{1}{0.03568}+\frac{37\ 466}{29.63^2} &= 28.02690583+\frac{37\ 466}{877.9369}\\ &= 28.02690583+42.67504874\\ &= 70.70195457\end{aligned}$$
which rounds to 70.70, assuming that the 1 is exact.

41.
$$\begin{aligned}875\text{ Btu} &= 875\text{ Btu}\times\frac{778.2\text{ ft}\cdot\text{lb}}{1\text{ Btu}}\times\frac{1.356\text{ J}}{1\text{ ft}\cdot\text{lb}}\\ &= 923{,}334.3\text{ J}\end{aligned}$$
which rounds to 923,000 J.

42.
$$\begin{aligned}18.4\text{ in} &= 18.4\text{ in}\times\frac{2.54\text{ cm}}{1\text{ in}}\times\frac{1\text{ m}}{100\text{ cm}}\\ &= 0.46736\text{ m}\end{aligned}$$
which rounds to 0.467 m.

43.
$$\begin{aligned}65\ \frac{\text{km}}{\text{h}} &= 65\ \frac{\text{km}}{\text{h}}\times\frac{1\text{ hr}}{3600\text{ s}}\times\frac{0.6214\text{ mi}}{1\text{ km}}\times\frac{5280\text{ ft}}{1\text{ mi}}\\ &= 59.2401333\ \frac{\text{ft}}{\text{s}}\end{aligned}$$
which rounds to 59 ft/s.

44. $12.25\ \frac{\text{g}}{\text{L}} = 12.25\ \frac{\text{g}}{\text{L}} \times \frac{28.32\ \text{L}}{1\ \text{ft}^3} \times \frac{1\ \text{kg}}{1000\ \text{g}} \times \frac{2.205\ \text{lb}}{1\ \text{kg}}$

$= 0.7649586\ \frac{\text{lb}}{\text{ft}^3}$

which rounds to 0.7650 lb/ft^3.

45. $225\ \text{hp} = 225\ \text{hp} \times \frac{550\ \text{ft} \cdot \text{lb/s}}{1\ \text{hp}} \times \frac{1.356\ \text{J}}{1\ \text{ft} \cdot \text{lb}} \times \frac{60\ \text{s}}{1\ \text{min}}$

$= 10068300\ \frac{\text{J}}{\text{min}}$

which rounds to $10100000 = 1.01 \times 10^7$ J/min.

46. $89.7\ \frac{\text{lb}}{\text{in}^2} = 89.7\ \frac{\text{lb}}{\text{in}^2} \times \frac{4.448\ \text{N}}{1\ \text{lb}} \times \left(\frac{1\ \text{in}}{2.54\ \text{cm}}\right)^2$

$= 61.8428917\ \frac{\text{N}}{\text{cm}^2}$

which rounds to 61.8 N/cm^2.

47. $a - 3ab - 2a + ab = -2ab - a$

48. $xy - y - 5y - 4xy = -3xy - 6y$

49. $6LC - (3 - LC) = 6LC - 3 + LC = 7LC - 3$

50. $-(2x - b) - 3(-x - 5b) = -2x + b + 3x + 15b = 16b + x$

51. $(2x - 1)(5 + x) = (2x)(5) + (2x)(x) + (-1)(5) + (-1)(x)$

$= 10x + 2x^2 - 5 - x$

$= 2x^2 + 9x - 5$

52. $(C - 4D)(D - 2C) = (C)(D) + (C)(-2C) + (-4D)(D) + (-4D)(-2C)$

$= CD - 2C^2 - 4D^2 + 8CD$

$= -2C^2 + 9CD - 4D^2$

53. $(x + 8)^2 = (x + 8)(x + 8)$

$= (x)(x) + (x)(8) + (8)(x) + (8)(8)$

$= x^2 + 8x + 8x + 64$

$= x^2 + 16x + 64$

54. $(2r - 9s)^2 = (2r - 9s)(2r - 9s)$

$= (2r)(2r) + (2r)(-9s) + (-9s)(2r) + (-9s)(-9s)$

$= 4r^2 - 18rs - 18rs + 81s^2$

$= 4r^2 - 36rs + 81s^2$

55. $$\frac{2h^3k^2-6h^4k^5}{2h^2k}=\frac{2h^3k^2}{2h^2k}-\frac{6h^4k^5}{2h^2k}$$
$$=h^{3-2}k^{2-1}-3h^{4-2}k^{5-1}$$
$$=-3h^2k^4+hk$$

56. $$\frac{4a^2x^3-8ax^4}{-2ax^2}=\frac{4a^2x^3}{-2ax^2}-\frac{8ax^4}{-2ax^2}$$
$$=-2a^{2-1}x^{3-2}+\frac{4\not{a}x^{4-2}}{\not{a}}$$
$$=4x^2-2ax$$

57. $$4R-[2r-(3R-4r)]=4R-[2r-3R+4r]$$
$$=4R-[6r-3R]$$
$$=4R-6r+3R$$
$$=7R-6r$$

58. $$-3b-[3a-(a-3b)]+4a=4a-3b-[3a-a+3b]$$
$$=4a-3b-[2a+3b]$$
$$=4a-3b-2a-3b$$
$$=2a-6b$$

59. $$2xy-\{3z-[5xy-(7z-6xy)]\}=2xy-\{3z-[5xy-7z+6xy)]\}$$
$$=2xy-\{3z-[11xy-7z]\}$$
$$=2xy-\{3z-11xy+7z\}$$
$$=2xy-\{10z-11xy\}$$
$$=2xy-10z+11xy$$
$$=13xy-10z$$

60. $$x^2+3b+[(b-y)-3(2b-y+z)]=x^2+3b+[b-y-6b+3y-3z)]$$
$$=x^2+3b+[-5b+2y-3z]$$
$$=x^2+3b-5b+2y-3z$$
$$=x^2-2b+2y-3z$$

61. $$(2x+1)(x^2-x-3)=(2x)(x^2)+(2x)(-x)+(2x)(-3)+(1)(x^2)+(1)(-x)+(1)(-3)$$
$$=2x^3-2x^2-6x+x^2-x-3$$
$$=2x^3-x^2-7x-3$$

62. $$(x-3)(2x^2+1-3x)=(x)(2x^2)+(x)(1)+(x)(-3x)+(-3)(2x^2)+(-3)(1)+(-3)(-3x)$$
$$=2x^3+x-3x^2-6x^2-3+9x$$
$$=2x^3-9x^2+10x-3$$

63. $$\begin{aligned}-3y(x-4y)^2 &= -3y(x-4y)(x-4y)\\ &= -3y[(x)(x)+(x)(-4y)+(-4y)(x)+(-4y)(-4y)]\\ &= -3y[x^2-4xy-4xy+16y^2]\\ &= -3y[x^2-8xy+16y^2]\\ &= -3x^2y+24xy^2-48y^3\end{aligned}$$

64. $$\begin{aligned}-s(4s-3t)^2 &= -s(4s-3t)(4s-3t)\\ &= -s[(4s)(4s)+(4s)(-3t)+(-3t)(4s)+(-3t)(-3t)]\\ &= -s[16s^2-12st-12st+9t^2]\\ &= -s[16s^2-24st+9t^2]\\ &= -16s^3+24s^2t-9st^2\end{aligned}$$

65. $$\begin{aligned}3p[(q-p)-2p(1-3q)] &= 3p[q-p-2p+6pq]\\ &= 3p[q-3p+6pq]\\ &= 18p^2q-9p^2+3pq\end{aligned}$$

66. $$\begin{aligned}3x[2y-r-4(x-2r)] &= 3x[2y-r-4x+8r]\\ &= 3x[2y+7r-4x]\\ &= 21rx-12x^2+6xy\end{aligned}$$

67. $$\begin{aligned}\frac{12p^3q^2-4p^4q+6pq^5}{2p^4q} &= \frac{12p^3q^2}{2p^4q}-\frac{4p^4q}{2p^4q}+\frac{6pq^5}{2p^4q}\\ &= \frac{6q^{2-1}}{p^{4-3}}-\frac{2\cancel{p^4q}}{\cancel{p^4q}}+\frac{3q^{5-1}}{p^{4-1}}\\ &= \frac{3q^4}{p^3}+\frac{6q}{p}-2\end{aligned}$$

68. $$\begin{aligned}\frac{27s^3t^2-18s^4t+9s^2t}{-9s^2t} &= \frac{27s^3t^2}{-9s^2t}-\frac{18s^4t}{-9s^2t}+\frac{9s^2t}{-9s^2t}\\ &= -3s^{3-2}t^{2-1}+\frac{2s^{4-2}\cancel{t}}{\cancel{t}}-\frac{\cancel{9s^2t}}{\cancel{9s^2t}}\\ &= 2s^2-3st-1\end{aligned}$$

69.
$$\begin{array}{r|l} & \quad 2x-5\\ x+6 & 2x^2+7x-30\\ & \underline{2x^2+12x}\\ & \quad -5x-30\\ & \quad \underline{-5x-30}\\ & \qquad\quad 0\end{array}$$

70.

$$\begin{array}{r} 2x-7 \\ 2x+7\overline{)4x^2+0x-41} \\ \underline{4x^2+14x} \\ -14x-41 \\ \underline{-14x-49} \\ 8 \end{array}$$

$$\frac{4x^2-41}{2x+7}=2x-7+\frac{8}{2x+7}$$

71.

$$\begin{array}{r} x^2-2x+3 \\ 3x-1\overline{)3x^3-7x^2+11x-3} \\ \underline{3x^3-x^2} \\ -6x^2+11x \\ \underline{-6x^2+2x} \\ 9x-3 \\ \underline{9x-3} \\ 0 \end{array}$$

72.

$$\begin{array}{r} w^2-w+4 \\ w-3\overline{)w^3-4w^2+7w-12} \\ \underline{w^3-3w^2} \\ -w^2+7w \\ \underline{-w^2+3w} \\ 4w-12 \\ \underline{4w-12} \\ 0 \end{array}$$

73.

$$\begin{array}{r} 4x^3-2x^2+6x \\ x+3\overline{)4x^4+10x^3+0x^2+18x-1} \\ \underline{4x^4+12x^3} \\ -2x^3+0x^2 \\ \underline{-2x^3-6x^2} \\ 6x^2+18x \\ \underline{6x^2+18x} \\ 0x-1 \end{array}$$

$$\frac{4x^4+10x^3+18x-1}{x+3}=4x^3-2x^2+6x-\frac{1}{x+3}$$

74.

$$\begin{array}{r} 4x^2-6x+2 \\ 2x+3\overline{)8x^3+0x^2-14x+3} \end{array}$$

$$\underline{8x^3+12x^2}$$

$$-12x^2-14x$$

$$\underline{-12x^2-18x}$$

$$4x+3$$

$$\underline{4x+6}$$

$$-3$$

$$\frac{8x^3-14x+3}{2x+3}=4x^2-6x+2-\frac{3}{2x+3}$$

75.

$$\begin{aligned} -3\{(r+s-t)-2[(3r-2s)-(t-2s)]\} &= -3\{r+s-t-2[3r-2s-t+2s)]\} \\ &= -3\{r+s-t-2[3r-t]\} \\ &= -3\{r+s-t-6r+2t]\} \\ &= -3\{-5r+s+t\} \\ &= 15r-3s-3t \end{aligned}$$

76.

$$\begin{aligned} &(1-2x)(x-3)-(x+4)(4-3x) \\ &=[(1)(x)+(1)(-3)+(-2x)(x)+(-2x)(-3)]-[(x)(4)+(x)(-3x)+(4)(4)+(4)(-3x) \\ &=[x-3-2x^2+6x]-[4x+-3x^2+16+-12x] \\ &=[-2x^2+7x-3]-[-3x^2-8x+16] \\ &=-2x^2+7x-3+3x^2+8x-16 \\ &=x^2+15x-19 \end{aligned}$$

77.

$$\begin{array}{r} y^2+5y-1 \\ 2y-1\overline{)2y^3+9y^2-7y+5} \end{array}$$

$$\underline{2y^3-1y^2}$$

$$10y^2-7y$$

$$\underline{10y^2-5y}$$

$$-2y+5$$

$$\underline{-2y+1}$$

$$4$$

$$\frac{2y^3+9y^2-7y+5}{2y-1}=y^2+5y-1+\frac{4}{2y-1}$$

78.

$$\begin{array}{r} 3x+4y \\ 2x-y\overline{)6x^2+5xy-4y^2} \end{array}$$

$$\underline{6x^2-3xy}$$

$$8xy-4y^2$$

$$\underline{8xy-4y^2}$$

$$0$$

79. $3x+1=x-8$

$2x=-9$

$x=-\frac{9}{2}$

80. $4y-3=5y+7$

$-y=10$

$y=-10$

81. $\frac{5x}{7}=\frac{3}{2}$

$2(5x)=3(7)$

$10x=21$

$x=\frac{21}{10}$

82. $\frac{2(N-4)}{3}=\frac{5}{4}$

$\frac{2N-8}{3}=\frac{5}{4}$

$4(2N-8)=3(5)$

$8N-32=15$

$8N=47$

$N=\frac{47}{8}$

83. $-6x+5=-3(x-4)$

$-6x+5=-3x+12$

$-3x=7$

$x=-\frac{7}{3}$

84. $-2(-4-y)=3y$

$8+2y=3y$

$y=8$

85. $2s+4(3-s)=6$

$2s+12-4s=6$

$-2s=-6$

$s=\frac{-6}{-2}$

$s=3$

86. $2|x|-1=3$

$$\begin{aligned} 2|x|&=4\\ |x|&=\frac{4}{2}\\ |x|&=2\\ x&=-2 \text{ and } 2 \end{aligned}$$

87. $3t-2(7-t)=5(2t+1)$

$$\begin{aligned} 3t-14+2t&=10t+5\\ 5t-14&=10t+5\\ -5t&=19\\ t&=-\frac{19}{5} \end{aligned}$$

88. $-(8-x)=x-2(2-x)$

$$\begin{aligned} -8+x&=x-4+2x\\ -8+x&=3x-4\\ -2x&=4\\ x&=-\frac{4}{2}\\ x&=-2 \end{aligned}$$

89. $2.7+2.0(2.1x-3.4)=0.1$

$$\begin{aligned} 2.7+4.2x-6.8&=0.1\\ 4.2x-4.1&=0.1\\ 4.2x&=4.2\\ x&=\frac{4.2}{4.2}\\ x&=1.0 \end{aligned}$$

90. $0.250(6.721-2.44x)=2.08$

$$\begin{aligned} 1.68025-0.610x&=2.08\\ -0.610x&=0.39975\\ x&=-\frac{0.39975}{0.610}\\ x&=0.655327868\\ x&=0.655 \end{aligned}$$

91. 60,000,000,000,000 bytes = 6×10^{13} bytes

92. 25,000 mi/h = 2.5×10^{4} mi/h

93. 15,400,000,000 km = 1.54×10^{10} km

94. 1.02×10^{9} Hz = 1,020,000,000 Hz

95. 2.53×10^{13} mi = 25,300,000,000,000 mi

96. $10^7\ \text{ft}^2 = 10{,}000{,}000\ \text{ft}^2$

97. $10^{-12}\ \text{W/m}^2 = 0.000000000001\ \text{W/m}^2$

98. $0.00000015\ \text{m} = 1.5 \times 10^{-7}\ \text{m}$

99. $1.5 \times 10^{-1}\ \text{Bq/L} = 0.15\ \text{Bq/L}$

100. $0.00000018\ \text{m} = 1.8 \times 10^{-7}\ \text{m}$

101.
$$V = \pi r^2 L$$
$$L = \frac{V}{\pi r^2}$$

102.
$$R = \frac{2GM}{c^2}$$
$$c^2 R = 2GM$$
$$G = \frac{c^2 R}{2M}$$

103.
$$P = \frac{\pi^2 EI}{L^2}$$
$$L^2 P = \pi^2 EI$$
$$E = \frac{L^2 P}{\pi^2 I}$$

104.
$$f = p(c-1) - c(p-1)$$
$$f = cp - p - cp + c$$
$$f - c = -p$$
$$p = c - f$$

105.
$$Pp + Qq = Rr$$
$$Qq = Rr - Pp$$
$$q = \frac{Rr - Pp}{Q}$$

106.
$$V = IR + Ir$$
$$IR = V - Ir$$
$$R = \frac{V - Ir}{I}$$

107.
$$d = (n-1)A$$
$$d = An - A$$
$$d + A = An$$
$$n = \frac{d + A}{A}$$

108.
$$\begin{aligned} mu &= (m+M)v \\ mu &= mv + Mv \\ mu - mv &= Mv \\ M &= \frac{mu - mv}{v} \end{aligned}$$

109.
$$\begin{aligned} N_1 &= T(N_2 - N_3) + N_3 \\ N_1 - N_3 &= N_2T - N_3T \\ N_2T &= N_1 - N_3 + N_3T \\ N_2 &= \frac{N_1 - N_3 + N_3T}{T} \end{aligned}$$

110.
$$\begin{aligned} Q &= \frac{kAt(T_2 - T_1)}{L} \\ QL &= kAt(T_2 - T_1) \\ T_2 - T_1 &= \frac{QL}{kAt} \\ -T_1 &= \frac{QL}{kAt} - T_2 \\ T_1 &= T_2 - \frac{QL}{kAt} \end{aligned}$$

111.
$$\begin{aligned} R &= \frac{A(T_2 - T_1)}{H} \\ HR &= AT_2 - AT_1 \\ AT_2 &= HR + AT_1 \\ T_2 &= \frac{HR + AT_1}{A} \end{aligned}$$

112.
$$\begin{aligned} Z^2 \quad 1 - \frac{\lambda}{2a} \quad &= k \\ Z^2 - \frac{Z^2\lambda}{2a} &= k \\ Z^2 - k &= \frac{Z^2\lambda}{2a} \\ 2a(Z^2 - k) &= Z^2\lambda \\ \lambda &= \frac{2aZ^2 - 2ak}{Z^2} \end{aligned}$$

113.
$$\begin{aligned} d &= kx^2[3(a+b) - x] \\ d &= kx^2[3a + 3b - x] \\ d &= 3akx^2 + 3bkx^2 - kx^3 \\ 3akx^2 &= d - 3bkx^2 + kx^3 \\ a &= \frac{d - 3bkx^2 + kx^3}{3kx^2} \end{aligned}$$

114.
$$\begin{aligned} V &= V_0[1+3a(T_2-T_1)] \\ V &= V_0[1+3aT_2-3aT_1] \\ V &= V_0+3aT_2V_0-3aT_1V_0 \\ 3aT_2V_0 &= V-V_0+3aT_1V_0 \\ T_2 &= \frac{V-V_0+3aT_1V_0}{3aV_0} \end{aligned}$$

115. $\dfrac{5.25\times 10^{13}\text{ bytes}}{6.4\times 10^{4}\text{ bytes}} = 8.203125\times 10^{8}$

which rounds to 8.2×10^8. The newer computer's memory is 8.2×10^8 larger.

116. $t = 0.25\sqrt{66} = 2.0310096$ s

which rounds to 2.0 s. It would take the person 2.0 s to fall 66 ft.

117. $\dfrac{0.553\text{ km}}{0.442\text{ km}} = 1.25113122$

which rounds to 1.25. The CN Tower is 1.25 times taller than the Sears tower.

118. $t = \left(\dfrac{4.8\times 10^{3}\text{ cells}}{2650}\right)^2 = (1.8113207)^2 = 3.280882876$ s

which rounds to 3.28 s. It would take the computer 3.28 s to check 4800 memory cells.

119.
$$\begin{aligned} \frac{R_1R_2}{R_1+R_2} &= \frac{(0.0275\ \Omega)(0.0590\ \Omega)}{0.0275\ \Omega+0.0590\ \Omega} \\ &= \frac{0.0016225\ \Omega^2}{0.0865\ \Omega} \\ &= 0.018757225\ \Omega \end{aligned}$$

which rounds to $0.0188\ \Omega$. The combined electric resistance is $0.0188\ \Omega$.

120.
$$\begin{aligned} 1.5\times 10^{11}\sqrt{\frac{m}{M}} &= 1.5\times 10^{11}\sqrt{\frac{5.98\times 10^{24}\text{ kg}}{1.99\times 10^{30}\text{ kg}}} \\ &= 1.5\times 10^{11}\sqrt{0.000003005} \\ &= 1.5\times 10^{11}(0.0017335) \\ &= 260\ 025\ 124.4\text{ m} \end{aligned}$$

which rounds to 2.6×10^8 m. The distance the space craft will be from the earth is 2.6×10^8 m.

121.
$$\begin{aligned} (x-2a)+3\text{ ft/yd}\cdot(x+2a) &= x-2a+3x+3(2a) \\ &= x-2a+3x+6a \\ &= 4x+4a \end{aligned}$$

The sum of their length is $4x + 4a$ ft.

122.
$$\begin{aligned} (Ai-R)(1+i)^2 &= (Ai-R)(1+i)(1+i) \\ &= (Ai-R)[(1)(1)+(1)(i)+(i)(1)+(i)(i)] \\ &= (Ai-R)[i^2+2i+1] \\ &= (Ai)(i^2)+(Ai)(2i)+(Ai)(1)+(-R)(i^2)+(-R)(2i)+(-R)(1) \\ &= Ai^3+2Ai^2+Ai-i^2R-2iR-R \end{aligned}$$

123. $4(t+h)-2(t+h)^2 = 4t+4h-2(t+h)(t+h)$

$$= 4t+4h-2[(t)(t)+(t)(h)+(h)(t)+(h)(h)]$$
$$= 4t+4h-2[t^2+2ht+h^2]$$
$$= 4t+4h-2t^2-4ht-2h^2$$
$$= -2t^2-2h^2-4ht+4t+4h$$

124. $\dfrac{k^2r-2h^2k+h^2rv^2}{k^2r} = \dfrac{k^2r}{k^2r}-\dfrac{2h^2k}{k^2r}+\dfrac{h^2rv^2}{k^2r}$

$$= \frac{\cancel{k^2}r}{\cancel{k^2}r}-\frac{2h^2}{k^{2-1}r}+\frac{h^2\cancel{r}v^2}{k^2\cancel{r}}$$
$$= 1-\frac{2h^2}{kr}+\frac{h^2v^2}{k^2}$$

125. $3\times 18\div(9-6)=54\div(3)=18$

$3\times 18\div 9-6=54\div 9-6=6-6=0$

Yes, the removal of the parentheses does affect the answer.

126. $(3\times 18)\div 9-6=54\div 9-6=6-6=0$

$3\times 18\div 9-6=54\div 9-6=6-6=0$

No, the removal of the parentheses does not affect the answer.

127. $x-(3-x)=2x-3$

$x-3+x=2x-3$

$2x-3=2x-3$

The equation is valid for all values of the unknown, so the equation is an identity.

128. $7-(2-x)=x+2$

$7-2+x=x+2$

$x+5=x+2$

$5=2$

The equation has no values of the unknown for which it is valid, so the equation is a contradiction.

129. **(a)** $2|2|-2|4|=4-8$

$=-4$

(b) $2|2-(-4)|=2|6|$

$=12$

130. For $a<0$, $|a|=-a$.

131. Given $3-x=0$,

$|3-x|+7=2x$

$-(3-x)+7=2x$

$-3+x+7=2x$

$4=x$

This is consistent with $3-x=0$, so $x=4$.

132. $|x-4|+6=3x$

$|x-4|=3x-6$

$x-4=3x-6$ or $-(x-4)=3x-6$

$2=2x$ $\quad$ $4-x=3x-6$

$x=1$ $\quad$ $10=4x$

and so the only possible solutions are

$x=1$ or $x=5/2$.

The first possibility, $x=1$, yields $|-3|+6=3$ or $9=3$, which is false.

The second possibility, $x=5/2$, yields $|-3/2|+6=15/2$ or $15/2=15/2$, which is true, and so the only solution is $x=5/2$.

133. $(x-y)^3=(x-y)(x-y)(x-y)$

$=(-(y-x))(-(y-x))(-(y-x))$

$=-(y-x)(y-x)(y-x)$

$=-(y-x)^3$

134. Generally, $(a \div b)\div c \neq a \div (b \div c)$

We demonstrate this using $a=8, b=4, c=2$:

$(8\div 4)\div 2 = 2\div 2 = 1$

$8\div(4\div 2) = 8\div 2 = 4$

Division is not associative.

135. $\dfrac{8\times 10^{-3}}{2\times 10^{4}} = 4\times 10^{-7}$

136. $\dfrac{\sqrt{4+36}}{\sqrt{4}} = \dfrac{\sqrt{(2)(2)(10}}{2} = \dfrac{2\sqrt{10}}{2} = \sqrt{10}$

137. $250 \text{ hp} = 250 \text{ hp} \times \dfrac{746.0 \text{ W}}{1 \text{ hp}} \times \dfrac{1 \text{ kW}}{1000 \text{ W}}$

$= 186.5 \text{ kW}$

This is rounded to 190 kW.

138. $32\ \dfrac{\text{lb}}{\text{in}^2} = 32\ \dfrac{\text{lb}}{\text{in}^2} \times \dfrac{4.448 \text{ N}}{1 \text{ lb}} \times \dfrac{1 \text{ in}}{2.54 \text{ cm}}^{2} \times \dfrac{100 \text{ cm}}{1 \text{ m}}^{2}$

$= 220,621.241\ \dfrac{\text{N}}{\text{m}^2}$

This is rounded to 220,000 N/m^2.

139. $110 \text{ N}\cdot\text{m} = 110 \text{ N}\cdot\text{m} \times \dfrac{1 \text{ lb}}{4.448 \text{ N}} \times \dfrac{1 \text{ ft}}{0.3048 \text{ m}}$

$= 81.1358787 \text{ ft}\cdot\text{lb}$

This is rounded to 81 foot pounds.

140. $1.2\times 10^{6}\ \frac{\text{A}}{\text{m}^2} = 1.2\times 10^{6}\ \frac{\text{A}}{\text{m}^2}\times \frac{1000\ \text{mA}}{1\ \text{A}}\times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^{2}$

$= 1.2\times 10^{5}\ \frac{\text{mA}}{\text{cm}^2}$

141. Let x = the cost of the first computer program.
Let x + \$72 = the cost of the second computer program.

$$x + (x + \$72) = \$190$$
$$2x + \$72 = \$190$$
$$2x = \$118$$
$$x = \frac{\$118}{2}$$
$$x = \$59$$

The cost of the first computer program is \$59, and the other program costs (\$59 + \$72) = \$131.
Check: \$59 + \$131 = 190

142. Let x = the cost to run the commercial on the first station.
Let x + \$1100 = the cost to run the commercial on the second station.

$$x + (x + \$1100) = \$9500$$
$$2x + \$1100 = \$9500$$
$$2x = \$8400$$
$$x = \frac{\$8400}{2}$$
$$x = \$4200$$

The cost of the run the commercial on the first station is \$4200, and the cost for the other station is (\$4200 + \$1100) = \$5300.
Check: \$4200 + \$5300 = \$9500

143. Let $2x$ = the amount of oxygen produced in cm^3 by the first reaction.
Let x = the amount of oxygen produced in cm^3 by the second reaction.
Let $4x$ = the amount of oxygen produced in cm^3 by the third reaction.

$$2x + x + 4x = 560\ \text{cm}^3$$
$$7x = 560\ \text{cm}^3$$
$$x = \frac{560\ \text{cm}^3}{7}$$
$$x = 80\ \text{cm}^3$$

The first reaction produces ($2 \times 80\ \text{cm}^3$) = $160\ \text{cm}^3$ of oxygen, the second reaction produces $80\ \text{cm}^3$ of oxygen, and the third reaction produces ($4 \times 80\ \text{cm}^3$) = $320\ \text{cm}^3$ of oxygen.
Check: $160\ \text{cm}^3 + 80\ \text{cm}^3 + 320\ \text{cm}^3 = 560\ \text{cm}^{3*}$

144. Let x = the speed that the river is flowing in mi/h.
Let x + 5.5 mi/h = the speed that the boat travels downstream.
Let $-x$ + 5.5 mi/h = the speed that the boat travels upstream.
The distance that the boat travelled is the same in both experiments. Distance = speed × time.

$$(x+5.5 \text{ mi/h})(5.0 \text{ h}) = (-x+5.5 \text{ mi/h})(8.0 \text{ h})$$
$$(5.0 \text{ h})(x)+(5.5 \text{ mi/h})(5.0 \text{ h}) = (8.0 \text{ h})(-x)+(5.5 \text{ mi/h})(8.0 \text{ h})$$
$$(5.0 \text{ h})(x)+(27.5 \text{ mi}) = (-8.0 \text{ h})(x)+(44 \text{ mi})$$
$$(13.0 \text{ h})x = 16.5 \text{ mi}$$
$$x = \frac{16.5 \text{ mi}}{13 \text{ h}}$$
$$x = 1.269230769 \text{ mi/h}$$

which rounds to 1.3 mi/h. The polluted stream is flowing at 1.3 mi/h.
Check:

$$(1.269230769 \text{ mi/h}+5.5 \text{ mi/h})(5.0 \text{ h}) = (-1.269230769 \text{ mi/h}+5.5 \text{ mi/h})(8.0 \text{ h})$$
$$(6.769230769 \text{ mi/h})(5.0 \text{ h}) = (4.2 \text{ mi/h})(8.0 \text{ h})$$
$$(33.8 \text{ mi}) = (33.8 \text{ mi})$$

145. Let x = the resistance in the first resistor in Ω.
Let x + 1200 Ω = the resistance in the second resistor in Ω.
Voltage = current × resistance. 2.4 μA = 2.3 × 10^{-6} A. 12 mV = 0.0120 V

$$(2.4\times10^{-6}\text{A})(x)+(2.4\times10^{-6}\text{A})(x+1200\ \Omega) = 0.0120 \text{ V}$$
$$(2.4\times10^{-6}\text{A})(x)+(2.4\times10^{-6}\text{A})(x)+(2.4\times10^{-6}\text{A})(1200\ \Omega) = 0.0120 \text{ V}$$
$$(4.8\ \times10^{-6}\text{A})(x)+(0.00288\text{V}) = 0.0120 \text{ V}$$
$$(4.0\times10^{-6}\text{A})(x) = 0.00912 \text{ V}$$
$$x = \frac{0.00912 \text{ V}}{4.8\times10^{-6}\text{A}}$$
$$x = 1900\ \Omega$$

The first resistor's resistance is 1900 Ω and the second resistor's is (1900 Ω + 1200 Ω) = 3100 Ω.
Check:

$$(2.4\times10^{-6}\text{A})(1900\ \Omega)+(2.4\times10^{-6}\text{A})(1900\ \Omega+1200\ \Omega) = 0.0120 \text{ V}$$
$$0.00456 \text{ V}+0.00744 \text{ V} = 0.0120 \text{ V}$$
$$0.0120 \text{ V} = 0.0120 \text{ V}$$

146. Let x = the concentration of the first pollutant in ppm.
Let $4x$ = the concentration of the second pollutant in ppm.

$$x+4x = 4.0 \text{ ppm}$$
$$5x = 4.0 \text{ ppm}$$
$$x = \frac{4.0 \text{ ppm}}{5}$$
$$x = 0.8 \text{ ppm}$$

The concentration of the first pollutant is 0.8 ppm, and the concentration of the second is (4 × 0.8 ppm) = 3.2 ppm.
Check:

$$0.8 \text{ ppm}+4(0.8 \text{ ppm}) = 4.0 \text{ ppm}$$
$$0.8 \text{ ppm}+3.2 \text{ ppm} = 4.0 \text{ ppm}$$
$$4.0 \text{ ppm} = 4.0 \text{ ppm}$$

147. Let x = the time taken in hours for the crew to build 250 m of road.
The crew works at a rate of 450 m/12 h, which is 37.5 m/h. Time = distance / speed.

$$x = \frac{250 \text{ m}}{37.5 \text{ m/h}}$$
$$x = 6.666666667 \text{ h}$$

which rounds to 6.7 h.

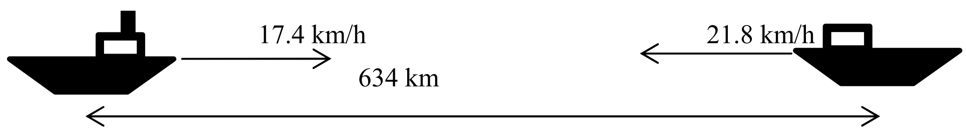
148. Let x = the amount of oil in L in the mixture.
Let $15x$ = the amount of gas in L in the mixture.

$$x + 15x = 6.6 \text{ L}$$
$$16x = 6.6 \text{ L}$$
$$x = \frac{6.6 \text{ L}}{16}$$
$$x = 0.4125 \text{ L}$$

which rounds to 0.41 L. There is 0.41 L of oil in the mixture and (15 × 0.41 L) = 6.2 L of gas.
Check:

$$0.4125 \text{ L} + 15(0.4125 \text{ L}) = 6.6 \text{ L}$$
$$0.4125 \text{ L} + 6.1875 \text{ L} = 6.6 \text{ L}$$
$$6.6 \text{ L} = 6.6 \text{ L}$$

149.



Let x = the time taken by the second ship in hours.
Let x + 2 h = the amount time taken by the first ship in hours.
The distance travelled adds up to 634km. Distance = speed × time.

$$21.8 \text{ km/h}(x) + 17.4 \text{ km/h}(x + 2 \text{ h}) = 634 \text{ km}$$
$$21.8 \text{ km/h}(x) + 17.4 \text{ km/h}(x) + 17.4 \text{ km/h}(2 \text{ h}) = 634 \text{ km}$$
$$39.2 \text{ km/h}(x) + 34.8 \text{ km} = 634 \text{ km}$$
$$39.2 \text{ km/h}(x) = 599.2 \text{ km}$$
$$x = \frac{599.2 \text{ km}}{39.2 \text{ km/h}}$$
$$x = 15.2857 \text{ h}$$

which rounds to 15.2 h. The ships will pass 15.2 h after the second ship enters the canal.
Check:

$$21.8 \text{ km/h}(15.2857 \text{ h}) + 17.4 \text{ km/h}(15.2857 \text{ h} + 2 \text{ h}) = 634 \text{ km}$$
$$333.23 \text{ km} + 300.77 \text{ km} = 634 \text{ km}$$
$$634 \text{ km} = 634 \text{ km}$$

150. Let x = the time take in h for the helicopter to travel from the pond to the fire.
Let 0.5 h $-x$ = the time take in h for the helicopter to travel from the fire to the pond.

30 min / 60 min/h = 0.5 h. The distance travelled by the helicopter is the same for both trips. Distance = speed × time.

$$105 \text{ mi/h}(0.5 \text{ h } - x) = 70 \text{ mi/h}(x)$$
$$52.5 \text{ mi } - 105 \text{ mi/h}(x) = 70 \text{ mi/h}(x)$$
$$52.5 \text{ mi } = 175 \text{ mi/h}(x)$$
$$x = \frac{52.5 \text{ mi}}{175 \text{ mi/h}}$$
$$x = 0.3 \text{ h}$$

which is reported as 0.30 h to two significant digits. It will take the helicopter 0.30 h to fly from the pond to the fire.
Check:

$$105 \text{ mi/h}(0.5 \text{ h } - 0.3 \text{ h}) = 70 \text{ mi/h}(0.3 \text{ h})$$
$$105 \text{ mi/h}(0.2 \text{ h}) = 70 \text{ mi/h}(0.3 \text{ h})$$
$$21 \text{ mi } = 21 \text{ mi}$$

151. Let x = the number of litres of 0.50% grade oil used.
Let 1000L $-\, x$ the number of litres of 0.75% grade oil used.

$$0.005(x) + 0.0075(1000 \text{ L} - x) = 0.0065(1000 \text{ L})$$
$$0.005(x) + 7.5 \text{ L} - 0.0075(x) = 6.5 \text{ L}$$
$$-0.0025(x) = -1.0 \text{ L}$$
$$x = \frac{-1.0 \text{ L}}{-0.0025}$$
$$x = 400 \text{ L}$$

It will take 400 L of the 0.50% grade oil and (1000 L – 400 L) = 600 L of the 0.75% grade oil to make 1000 L of 0.65% grade oil.
Check:

$$0.005(400 \text{ L}) + 0.0075(1000 \text{ L} - 400 \text{ L}) = 0.0065(1000 \text{ L})$$
$$2 \text{ L} + 4.5 \text{ L} = 6.5 \text{ L}$$
$$6.5 \text{ L} = 6.5 \text{ L}$$

152. Let x = the amount of rock containing 72 L/Mg of oil .
Let $18000 - x$ = the remaining amount of rock containing 150 L/Mg of oil.

$$(72 \text{ L/Mg})(x) + (150 \text{ L/Mg})(18000 \text{ Mg} - x) = (120 \text{ L/Mg})(18000 \text{ Mg})$$
$$72 \text{ L/Mg}(x) + 2700000 \text{ L} - 150\text{L/Mg}(x) = 2160000 \text{ L}$$
$$-78 \text{ L/Mg}(x) = -540000 \text{ L}$$
$$x = \frac{-540000 \text{ L}}{-78 \text{ L/Mg}}$$
$$x = 6923.07692 \text{ Mg}$$

which rounds to 6900 Mg. It will take 6900 Mg of 72 L/Mg rock and 11100 Mg of 150 L/Mg rock to make the 18000 Mg of 120 L/Mg rock.
Check:

$$(72 \text{ L/Mg})(6923.07692 \text{ Mg}) + (150 \text{ L/Mg})(18000 \text{ Mg} - 6923.07692 \text{ Mg}) = (120 \text{ L/Mg})(18000 \text{ Mg})$$
$$498461.538 \text{ L} + 2700000 \text{ L} - 1038461.538 \text{ L} = 2160000 \text{ L}$$
$$2160000 \text{ L} = 2160000 \text{ L}$$

153. Let x = the area of space in ft^2 in the kitchen and bath.

$$\frac{\text{ft}^2 \text{ of tile in the house}}{\text{ft}^2 \text{ in the house}} = 0.25$$

$$\frac{x + 0.15(2200 \text{ ft}^2)}{(x + 2200 \text{ ft}^2)} = 0.25$$

$$x + 330 \text{ ft}^2 = 0.25(x) + (0.25)(2200 \text{ ft}^2)$$

$$x + 330 \text{ ft}^2 = 0.25(x) + 550 \text{ ft}^2$$

$$0.75x = 220 \text{ ft}^2$$

$$x = \frac{220 \text{ ft}^2}{0.75}$$

$$x = 293.33333333 \text{ ft}^2$$

which rounds to 290 ft^2. The kitchen and bath area is 290 ft^2.

Check:

$$\frac{293.33333333 \text{ ft}^2 + 0.15(2200 \text{ ft}^2)}{(293.33333333 \text{ ft}^2 + 2200 \text{ ft}^2)} = 0.25$$

$$\frac{623.3333333 \text{ ft}^2}{2493.3333333 \text{ ft}^2} = 0.25$$

$$0.25 = 0.25$$

154. Let x = the number of grams of 9-karat gold.
Let 200 g – x = the number of grams of 18-karat gold. 9-karat gold is 9/24 gold = 0.375, 18-karat gold is 18/24 gold= 0.75, and 14-karat gold is 14/24 gold = 0.583333333.

$$0.375(x) + 0.75(200 \text{ g} - x) = 0.583333333(200 \text{ g})$$

$$0.375(x) + 150 \text{ g} - 0.75(x) = 116.6666666 \text{ g}$$

$$-0.375(x) = -33.3333334 \text{ g}$$

$$x = \frac{-33.3333334 \text{ g}}{-0.375}$$

$$x = 88.88888907 \text{ g}$$

which rounds to 89 g. There is 89 g of 9-karat gold and (200 g – 89 g) = 111 g of 18-karat gold needed to make 200 g of 14-karat gold.

Check:

$$0.375(88.88888907 \text{ g}) + 0.75(200 \text{ g} - 88.88888907 \text{ g}) = 0.583333333(200 \text{ g})$$

$$33.3333334 \text{ g} + 83.3333332 \text{ g} = 116.6666666 \text{ g}$$

$$116.6666666 \text{ g} = 116.6666666 \text{ g}$$

155.

$$P = P_0 + P_0 rt$$

$$P - P_0 = P_0 rt$$

$$r = \frac{P - P_o}{P_0 t}$$

$$r = \frac{\$7625 - \$6250}{\$6250(4.000 \text{ years})}$$

$$r = \frac{\$1375}{25\ 000}$$

$$r = 0.055$$

The rate is equal to 5.500%.

On the calculator type:

$(7625 - 6250)/(6250 \times 4.000)$